UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY Preliminary Exam

August 10, 2012

9:00 am - 2:00 pm

INSTRUCTIONS

- **Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers,.** To complete the label, write:
  - (1) **your assigned number**
  - (2) the **number of the question you are answering**
  - (3) the **position of the page in the sequence** of pages used to answer the questions.

Example:	
MACRO THEORY	8/10/12
ASSIGNED # ____________	
Qu # __1__ (Page __2__of _4_):	

- **Do not answer more than one question on the same page !**
  When you start a new question, start a new page.
- **DO NOT write your name anywhere on your answer sheets!**
  After the examination, the question sheets and answer sheets will be collected.
- **Please DO NOT WRITE on the question sheets.**
- **Each question counts equally.**
- **Answer all questions.**
- **Answers will be penalized for extraneous material; be concise**
- **You are not allowed to use notes, books, calculators, or colleagues.**
- **Do NOT use colored pens or pencils**
- **There are 5 pages in the exam, including these instruction pages – please make sure you have all of them.**

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.
- All scratch paper, unused tablet paper, and exams are to be turned in after the exam. Your proctor will give you directions, listen to your proctor.
- Good luck!

1. Does the First Welfare Theorem hold in overlapping generations models with production and capital accumulation? If your answer is yes, prove it. If your answer is no, present a counterexample. (100 points)

Question 2: 100 total points.

A. (40 points) Consider a variation on the basic (McCall) sequential search model in which there is wage growth. Agents are risk neutral and seek to maximize:

$$E\sum_{t=0}^{\infty}\beta^t y_t$$

where y_t is income in period t, which comes either from work or unemployment benefits, and $0 < \beta < 1$. Suppose that there are no separations and each unemployed worker is sure to receive an offer upon searching. If the wage offer is w in the first period, then the wage is $w_t = \phi^t w$ after t periods on the job, where $\phi > 1$ and $\phi\beta < 1$. The initial wage offer is drawn from a constant distribution $F(w)$. Unemployed workers earn a constant benefit of b.

(a) Write down an unemployed worker's Bellman equation and characterize his optimal decision strategy.

(b) Suppose that there are two economies $i = 1, 2$ that differ in their wage growth rates, with $\phi_1 > \phi_2$. (Both ϕ_i still satisfy $1 < \phi_i < 1/\beta$.) How do the decision strategies differ across economies?

B. (60 points) Suppose that a representative agent has preferences:

$$E_0\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\gamma}}{1-\gamma}$$

over the single nonstorable consumption good ("fruit"), where $\gamma > 0$. Her endowment of the good is governed by a Markov process with transition function $F(x', x)$.

(a) Define a recursive competitive equilibrium with a market in claims to the endowment process ("trees").

(b) When the consumer has logarithmic utility, $u(c) = \log c$, what is the equilibrium price/dividend ratio of a claim to the entire consumption stream? How does it depend on the distribution of consumption growth?

(c) Suppose there is news at time t that future consumption will be higher. How will prices respond to this news? How does this depend on the consumer's preferences (which are CRRA, but not necessarily log)? Interpret your results.

(d) Suppose that the endowment process is characterized by lognormal growth. That is, $x_{t+1} = x_t \exp(\xi_{t+1})$ where $\xi_t \sim N(\mu, \sigma^2)$ i.i.d. What is one-period risk free interest rate? How does it depend on the preference parameter γ? Interpret your results.

3) (100 points) Each agent goes through 3 periods of life: kid, young parent, old parent (and then dies). Each kid is born with a stochastic ability a. The distribution of the kid's ability is a function of the parent's ability, i.e. $a' \sim A(a'|a)$. Each young parent gives birth to one kid and makes decisions for him. A parent (or a grand-parent) cannot purchase insurance against the ability of his children.

The young parent invests e units of consumption goods and n units of time in his *own* human capital accumulation ("on-the-job training"), e_k units of consumption goods in his kid's human capital. Assume that the total amount of time endowed to each individual is 1 in each period. Human capital evolves according to

$$h'_o = a(nh)^{\gamma_1} e^{\gamma_2} + (1-\delta)h$$
$$h'_k = a_k(h)^{\gamma_1} e_k^{\gamma_2} + (1-\delta)h$$

where h is the human capital of the parent when he is a young adult, h'_o is the human capital of the parent when he gets old, and h'_k is the human capital of the child when he grows up and becomes a young parent.

The amount the young adult earns in the second period depends on the wage rate, w, the stock of human capital, the choice of leisure, l and a labor market "luck" shock ε.The young parent, for instance, earns the amount $wh(1-n-l)\varepsilon$. The luck shock ε is drawn from the distribution Φ and is iid across generations.

The young parent saves s for his old age when young, starts off the second period with inter-vivos transfers (transfers made while parent is alive) of i and begins the third period with bequests b from his parent. Assume that the young parent makes decisions *after* she receives her inter-vivos transfer from his parent and that the old parent makes decisions after he has receives the bequest b. Preferences are defined as

$$u(c_y, l_y) + \beta \mathbb{E}\left\{u(c'_o, 1) + \theta V'\right\},$$

where c_y is the consumption of the young parent, c'_o is the consumption of the old parent, and V' is the lifetime utility of her child after he grows up and the expectation operator is over future abilities. θ is the weight the parent puts on her child's utility. Assume that we are in a stationary equilibrium.

1. Let $V(\cdot)$ be the value for the young parent and $J(\cdot)$ for the old parent, formulate the Bellman equations for the young and old parent. Be careful to clarify the states and controls. (15 points)

2. Briefly outline an argument for why the pair of Bellman equations is a contraction. (10 points)

3. Briefly outline an argument that demonstrates that the value function is concave (Asssume that it exists and is given by the unique solution to the Bellaman's equation above). Also outline an argument that demonstrates that the Value function $V(\cdot)$ is differentiable. (10 points)

4. Derive all the first order conditions and the envelope conditions. Interpret the Euler equation(s). (20 points)

5. Compare the FOCs for e and e_k. Do they look different in structure? Why? (5 points)

6. Consider the General Equilibrium version of the model presented above. That is, apart from a continuum of consumers, we also have a standard Neoclassical firm that rents capital and human capital from consumers. Now suppose that markets are complete. What do the optimal choices of savings and investment in children look like? Explain how this differs from the incomplete market allocation. (10 points)

7. Imagine in this economy we considered a government that levies a tax on earnings (so that earnings for the young are $(1\text{-}\tau)wh(1-n-l)\varepsilon$, where τ is the tax rate) and rebates it back to (young and old) households in a lumpsum fashion. Would (all) consumers want this tax-transfer scheme starting from no governmental intervention whatsoever? Why? (10 points)

8. Consider an economy with incomplete markets such as the one presented above. Assume that in the steady state both physical and human capital are higher than in the steady state of the complete market case. This implies that output is higher and consumption is higher. Seems like the individual is better off with incomplete markets. What is wrong with this logic? (10 points)

9. What happens to consumption when a given individual retires (old)? Does it increase or decrease relative to consumption when young and what does this increase depend upon? (10 points)