Endogenous Liquidity and Capital Reallocation*

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Abstract

We study economies where firms acquire capital in primary markets, then, after productivity shocks, re-trade it in secondary markets. Our secondary markets incorporate bilateral trade, potentially with search, bargaining and liquidity frictions. A novel feature is to distinguish between full or partial sales (one firm gets all or some of the other's capital). We show both exhibit interesting long- and short-run patterns in the data. Outcomes depend on monetary and credit conditions: more partial sales occur when liquidity is tight. Analytically and numerically, we study steady-state and business-cycle implications, plus the impact of search, taxation, and persistence in firm-specific shocks.

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Used equipment and structures sometimes trade unbundled in that firm 1 buys a machine or building from firm 2, but firm 2 continues to exist. At other times, firm 1 buys firm 2 and thereby gets to own all of firm 2's capital. Jovanovic and Rousseau (2002).

1 Introduction

This paper studies economies where firms first acquire capital in centralized primary markets, as in standard growth theory, then, after idiosyncratic productivity shocks are realized, they reallocate it in decentralized secondary markets. In the interest of realism and generality our decentralized markets incorporate bilateral trade with search, matching, bargaining and liquidity frictions, although we can consider special cases where each of these is suppressed. A novel feature is that we distinguish between full sales, sometimes called acquisitions or buyouts, where in a bilateral deal the buyer gets all of the capital of the seller, and partial sales, where the buyer gets some but not all. Under constant returns to scale it is socially efficient for the firm with higher productivity to get all the capital, but that does not always happen in equilibrium. Our goal is to study how this depends on parameters and policies, and to analyze the macroeconomic implications.

We first document that the two types of reallocation (full and partial sales) exhibit interesting empirical patterns. At the business cycle frequency, the ratio of full sales to total capital expenditure is procyclical while the ratio of partial sales to capital expenditure is countercyclical. In the longer run, over several decades, the ratio of full sales to capital expenditure has increased while the ratio of partial sales to capital expenditure has decreased. Then, given that 42% of full sales are facilitated by cash or cash-equivalent payments in the data (Thomson Reuters M&A Database, 1971-2018), we examine the relationship between reallocation and the cost of liquidity, which as discussed below is measured by inflation. In the longer run, full sales decrease while partial sales increase with inflation, but at the business cycle frequency the pattern is reversed.

We then develop a formal model of reallocation, where the mix between full and partial sales is determined endogenously and depends on monetary and fiscal policy. Theory predicts that higher inflation raises the cost of liquidity, decreasing full sales, increasing partial sales and, since full sales are larger, decreasing total reallocation. This is consistent with the long-run facts. Then, to get full sales to increase while partial sales decrease with inflation at business cycle frequencies, we introduce shocks to credit conditions. Easier credit increases total reallocation, increases full sales, decreases partial sales, plus it reduces firms' demand for money and that leads to a short-term jump in inflation. Thus, with credit shocks at the business cycle frequency total reallocation can be procyclical and move with inflation while partial sales are countercyclical.

While the model is designed to be consistent with these observations, it also provides a framework to study various other issues. For one, it provides insights into how frictions related to search, matching and bargaining affect trade in secondary markets and the impact on the macroeconomy. For another, it provides insights into how fiscal policy affects primary and secondary capital markets, and in particular we solve for the optimal capital tax (actually, subsidy, since it is negative) analytically and numerically. Also, while our baseline model assumes i.i.d. firm-specific productivity shocks, we also solve it with persistent firm-specific shocks, to study how persistence affects firms' capital and liquidity positions. One finding is that the responses of aggregate variables to aggregate shocks do not depend much on whether the idiosyncratic shocks are i.i.d. or persistent.

As motivation for studying capital reallocation, in general, note that efficiency requires not only getting the right amount of investment over time, but getting capital into the hands of those best able to use it at any point in time. While macroeconomists traditionally concentrated on the former, the latter is now receiving much attention. Idiosyncratic shocks make capital flow from lower- to higher-productivity firms, in theory, and in data (Maksimovic and Phillips 2001; Andrade et al. 2001; Schoar 2002). It is important to emphasize that the ease with which assets can be retraded on secondary markets affects investment primary markets for capital, as it does for, e.g., houses, cars or financial assets (Harrison and Kreps 1978; Lagos and Zhang 2019). But the channel is subtle: a well-functioning secondary market encourages primary investment since, if you have more capital than you need, it is relatively easy to sell; it also discourages primary investment because, if you want more capital than you have, it is relatively easy to buy. We show precisely how the net results depend on parameters.

As additional motivation, several studies document that reallocation is sizable, with purchases of used capital between 25% and 33% of total investment (Eisfeldt and Rampini 2006; Cao and Shi 2016; Dong et al. 2016; Cui 2017; Eisfeldt and Shi 2018). Indeed, these may be underestimates, since they ignore small firms and those that are not publicly traded, plus they neglect mergers, and include purchases but not rentals. Studies also document and discuss the relevance of several stylized facts: reallocation is procyclical while capital mismatch is countercyclical (Eisfeldt and Rampini 2006; Cao and Shi 2016); productivity dispersion is countercyclical (Kehrig 2015); the price of used capital is procyclical (Lanteri 2016); and the ratio of spending on used capital to total investment is procyclical (Cui 2017). Our model can account for these observations.

As for frictional reallocation, many authors argue that secondary capital markets are far from the perfectly competitive ideal (Gavazza 2010,2011*a,b*; Kurman 2014; Ottonello 2015; Kurmann and Rabinovitz 2018; Horner 2018). Imperfections include financial constraints, difficulties in finding the right counterparty, holdup problems, and asymmetric information. We somewhat downplay adverse selection (see, e.g., Li and Whited 2015 or Drenik et al. 2019) to concentrate on other issues. Thus, our secondary markets features bilateral trade and bargaining, as in search theory.[1] In addition, it features the use of assets to facilitate payments, as in monetary economics. Whether buyers get some or all of sellers' capital thus depends not just on productivity, but on liquidity. While explicit modeling of liquidity is missing from most previous studies of capital reallocation, Buera et al. (2011), Moll (2014) and others argue that financial frictions are important, even if self financing mitigates the problem somewhat, and we want to capture that.

To say more about our approach to liquidity, we do not narrowly conceive of money as currency, but any asset that is widely accepted as a payment instrument, or can be converted into something that is accepted with little cost or delay. Actual economies have

[1] In comparing models of capital with and without search, Ottonello (2015) finds the former fit the facts better and entail more interesting propagation. Horner (2018) shows vacancy rates for industrial, retail and office space resemble unemployment data, suggesting that search is as relevant for that kind of capital as it is for labor, and finds disperse rents on similar structures, inconsistent with frictionless markets. For aircraft, Pulvino (1998), Gilligan (2004) and Gavazza 2011*a,b*) find used sales are thrice new, while Gavazza (2011a) shows prices vary inversely with search time, and market thickness affects trading frequency, average utilization, utilization dispersion, average price and price dispersion. This work also emphasizes *specificity* – capital is often customized to a firm. This all suggests search and matching are relevant.

a spectrum of assets with varying degrees of acceptability and rates of return, implying there must be a tradeoff between these attributes. Research on the microfoundations of monetary economics tries to analyze this tradeoff explicitly, e.g., Kiyotaki and Wright (1989) is an early attempt to formalize it in a relatively clean environment, although one that is far too stylized for this paper, which is an exercise in macroeconomics.

The essence of macro is aggregation. In standard models there are two uses of generic output: consumption and investment. Similarly, there are typically two uses of time: work and leisure. (There are exceptions, like macro models with home production that have three uses for output and three uses for time, but that does not diminish the general point.) In this spirit, our benchmark model has two assets: capital; and what we call money, which is used to pay for capital in the secondary market, although some credit can also be used. In reality, while cash may be the most liquid asset, there are substitutes including bank deposits. Consistent with this, we incorporate banking as in Berentsen et al. (2007), which is useful in both theory and calibration.

But the key idea is that inflation represents the cost of liquidity because it lowers the return on the most liquid asset, and in equilibrium that lowers the return on other liquid assets, thus reducing overall liquidity and hindering reallocation. As Wallace (1980) put it: "Of course, in general [inflation] is not a tax on all saving. It is a tax on saving in the form of money. But it is important to emphasize that the equilibrium rate-of-return distribution on the equilibrium portfolio does depend on [inflation]... The models under consideration here imply that the higher the [inflation rate] the less favorable the terms of trade – in general, a distribution – at which present income can be converted into future income." While our baseline model has only money and capital, Appendix C adds a liquid real assets to show explicitly how this works.

In what follows, Section 2 discusses in more detail the above-mentioned facts as well as some microeconomic evidence. Sections 3 and 4 present the model and analytic results. Sections 5 and 6 contain the quantitative exercises. Section 7 concludes.[2]

[2] As for the literature, in addition to those mentioned above, relevant papers include Ramey and Shapiro (1998, 2001), Hsieh and Klenow (2009), Asker et al. (2014), Midrigan and Xu (2014), Cooper and Schott (2016), Ai et al. (2016), David et al. (2016), Lanteri and Gavazza (2019) and David and Venkateswaran (2019). Cui (2017) and Wright et al. (2020) discuss other papers, including some with capital and money, e.g. Shi (1999*a,b*), Shi and Wang (2006) and Aruoba et al. (2011).

2 Evidence

We use US data from 1971 to 2018 (details are in Appendix A). Capital reallocation is from COMPUSTAT, where there is information on full and partial sales, plus total capital expenditures, for each firm. This is summed to obtain an annual aggregate series on full sales, partial sales, and capital expenditures. Reallocation is defined as full plus partial sales. We focus on the reallocation-to-expenditures ratio, called the R share, and the partial-sales-to-reallocation ratio, called the P share. The former captures the importance of reallocation in total investment, and the latter the importance of partial sales in reallocation. In the early part of the sample the R share varies a lot, but after 1984 it stabilizes, fluctuating around 30%. Similarly, early in the sample the P share was around 70%, but it stabilizes after 1984, fluctuating around 31%.[3]

As discussed above, we entertain the possibility that liquidity plays a role in secondary capital markets and use inflation to capture its cost. We also tried the using the nominal T-bill and AAA corporate bond rates, and the results are similar, which is not surprising as inflation and nominal interest rates move together. Figure 1 (all Figures are at the end) shows the R and P shares vs inflation, with different panels using the raw data, plus the trend and cyclical components after applying a band-pass filter.

In the longer run, when inflation is high, firms spend less on used capital relative to total investment, and in fact spend less on both, while within reallocation there are more partial sales. Given full sales are twice partial sales, when inflation rises the fall in reallocation is mainly driven by full sales. Of course other secular changes could have affected reallocation, and the fall in inflation may or may not have resulted from monetary policy, but in any case in the longer run lower inflation is associated with more full sales and fewer partial sales. However, at business cycle frequencies the relationships are reversed. A plausible explanation involves credit conditions: easier credit leads to more full and fewer partial sales, plus it reduces money demand, which puts upward pressure on the price level and thus raises inflation in the short run.

In light of this, we construct a proxy for credit conditions using aggregate firm debt (see

[3] Because the R and P shares look relatively stationary after 1984, in the calibration below we start the sample then, despite starting here in 1971, but the conclusions do not depend on this.

Appendix A) in 2012 dollars. Figure 2 shows the cyclical components of debt, investment, the R share, the P share, and output. It is evident that debt is procyclical, as is the R share, while the P share is countercyclical. Therefore, when credit conditions ease firm debt goes up, part of which is used to fund reallocation, which is why full sales go up and partial sales down while total reallocation goes up.

Notice reallocation needs to be more volatile than investment to get a procyclical R share. As Table 1 shows, the primary capital market (investment) and secondary market (reallocation) comove. These business cycle statistics are consistent with the above discussion and suggest money demand and credit conditions are important over the cycle.[4]

Table 1: Business Cycle Correlations

	Debt	R share	P share	Investment	Inflation	Output
Debt	1	0.52(0.57)	-0.45(-0.57)	0.47(0.77)	0.18(0.46)	0.59(0.79)
R share	-	1	-0.77(-0.89)	0.62(0.73)	0.24(0.48)	0.64(0.68)
P share	-	-	1	-0.50(-0.69)	-0.14(-0.31)	-0.53(-0.68)
Investment	-	-	-	1	0.43(0.69)	0.96(0.97)
Inflation	-	-	-	-	1	0.37(0.59)
Output	-	-	-	-	-	1
Relative std. dev.	1.19	5.88	8.68	3.74	1.93	1

Note: the numbers in brackets are associated with NBER recession dates only.

Next, we use the disaggregated data in COMPUSTAT to present two pieces of micro evidence. First, we show money holdings have a positive effect on full buyouts and a negative or insignificant effect on partial buyouts. (In terms of labels, we usually use full and partial sales, but when the focus is on the firm acquiring capital it seems better to say full and partial buyouts, so we do so here.) Second, we examine how inflation impacts firms' money holdings.

First, we regress full buyouts on liquidity as measured by holdings of cash or cash equivalent, the latter consisting of assets that are readily converted to cash. Two outcomes are considered. The first is a binary variable that equals 1 if a firm engages in a full buyout this year, and 0 otherwise, which captures the extensive margin. (We use a linear probability model, but a logistic regression gives similar results.) The second ap-

[4]Notice that all the relationships are stronger if we look at recession dates only, suggesting that money demand plays a bigger role during downturns.

proach is to examine full buyout expenditure. For this we take logs, and focus on firms engaging in a full buyout in a given year, which captures the intensive margin. Importantly, for our purposes it does not matter whether money holdings cause full buyouts or the anticipation of full buyouts leads to more money holdings – either one means that cash facilitates reallocation.

For both outcomes, we control for variables that may affect full buyouts and money holdings, such as earnings before interest and taxes (EBIT), total assets, and the leverage ratio calculated as short-term liabilities over shareholder equity (SEQ). All the dependent variables are lagged by one period to avoid simultaneity problems. We also include year or year-industry fixed effects (FE) as controls, defined using the first two digits of SIC codes. All variables except the leverage ratio are in logs, and normalized by total assets of the firm (results are similar if we deflate by the CPI).

In Table 2, the first three columns give the probability of a full buyout. The first column includes only firm FE, the second includes firm and year FE and the third includes firm and year-industry FE. In all cases, money holdings have a significantly positive effect on the probability of a full buyout, with a 1% increase in cash raising the probability by around 0.00019 in levels. As the average probability of an full sale is around 0.21, this means a 1% increase in cash increases the full sale probability by close to 0.1%. EBIT has significant positive effects on full buyouts. Total assets have a positive effect and leverage ratios a negative effect. The last three columns of Table 2 report results on full buyout spending, with a 1% increase in money held leading to a 0.2% increase in spending, which is sizeable. Again EBIT has a positive effect on full buyout spending. We also ran dynamic panel regressions to account for the possibility that full buyouts may be persistent; the results are similar, and the coefficients on lagged buyouts are small and insignificant. All of this indicates that liquidity, as measured by holdings of cash or cash equivalent, encourages full buyouts.

Now consider partial buyouts. One downside of the COMPUSTAT data is that we cannot identify the buyer of each PPE purchase and hence cannot assess how buyers' cash matters. However, we can aggregate the data into industry level (defined by the first two digits of SIC) and investigate how cash held in an industry affects partial sales. If the

Table 2: Full Sales and Money Holdings

	Prob	Prob	Prob	Spending	Spending	Spending
Money Holdings	0.018***	0.019***	0.018***	0.190***	0.199***	0.200***
	(0.001)	(0.001)	(0.001)	(0.012)	(0.011)	(0.012)
EBIT	0.024***	0.027***	0.028***	0.267***	0.274***	0.264***
	(0.002)	(0.002)	(0.002)	(0.018)	(0.018)	(0.019)
Asset	0.082***	0.063***	0.066***	-0.317***	-0.384***	-0.383***
	(0.003)	(0.003)	(0.004)	(0.017)	(0.021)	(0.022)
Leverage	-0.001**	-0.001**	-0.001*	-0.005	-0.005	-0.005*
	(0.000)	(0.000)	(0.000)	(0.003)	(0.003)	(0.003)
Firm FE	Y	Y	Y	Y	Y	Y
Year FE		Y			Y	
Year-Industry FE			Y			Y
N	126354	126354	126354	37175	37175	37175

Note: Robust standard errors are in brackets and are clustered at firm level.
$p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

sales of PPE occur mostly within industry, i.e. firms mostly buy used PPE from other firms in the same industry, then evidence from an industry-level analysis should be similar to a firm-level analysis. Table 3 shows the results.

As before, purchases of PPE, full buyouts, money held, sales, assets are in log terms and normalized by total assets. All the dependent variables are values at the end of the last period, and all regressions include industry FE. The first column reports the effect of cash on purchases of PPEs without year FEs. The coefficient on cash held is negative and significant at the 5% level. The second column adds the year FE, and the coefficient is still negative but insignificant. We also include the results on full buyouts, and again cash has a significantly positive effect. The coefficient after introducing year FE is comparable to the firm-level regression. Therefore, it is plausible that capital reallocation happens mostly within an industry. Putting these pieces together, the conclusion is that money held has a positive effect on full buyouts, significant both statistically and economically, and an insignificant effect on partial buyouts.

Next consider how liquidity costs impact money holdings, shown in Table 4. In all regressions we control for firm FE. The first column is a static panel regression. It suggests

Table 3: Money Holdings on Partial and Full Sales Normalized by Assets

	PPE	PPE	Full Sales	Full Sales
Money Holdings	-0.255**	-0.051	0.425***	0.271***
	(0.129)	(0.103)	(0.080)	(0.076)
Sales	-0.170	-0.245**	0.224***	0.375***
	(0.121)	(0.094)	(0.097)	(0.085)
Log Asset	-0.505***	-0.094	0.057	-0.224***
	(0.084)	(0.091)	(0.075)	(0.070)
Leverage	0.234*	0.134	-0.108	0.167
	(0.133)	(0.112)	(0.220)	(0.168)
Year FE		Y		Y
N	2662	2662	2555	2555

Note: Robust standard errors are in brackets and are clustered at firm level.
$p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

that a 1% increase in the level of inflation reduces money held by more than 3.3%. However, money holdings today may also depend on previous money holdings, and to allow for that we estimate a dynamic panel model using the Arellano-Bond (1990) estimator, shown in the second column. The coefficient on past cash is about 0.24 and highly significant, while the effect of inflation is somewhat smaller but still significant both statistically and economically. As in our sample inflation decreases and money holdings increase over time, negative coefficients on inflation may result from the trends. To partly address this we use a Band-Pass filter to remove the trend component of inflation. The last two columns in Table 4 repeat the regressions using the cyclical component. Coefficients on inflation remain negative and highly significant, and in fact the magnitude is larger.

While more could always be done, the micro and macro results presented above suffice to motivate the study of both types of capital reallocation, taking into account liquidity and its cost measured by inflation, plus credit conditions.

3 Model

A $[0, 1]$ continuum of ex ante homogeneous agents live forever in discrete time. As shown in Figure 3, at each date t they interact in two distinct markets that convene sequentially:

Table 4: Money Holdings and Liquidity Cost

Money Holding				
Inflation	-3.348***	-1.939***		
	(0.122)	(0.163)		
Inflation (Cycle)			-4.020***	-2.667***
			(0.268)	(0.196)
Sales	0.195***	0.083***	0.185***	0.083***
	(0.005)	(0.005)	(0.005)	(0.005)
Log Asset	-0.122***	-0.096***	-0.114***	-0.081***
	(0.002)	(0.016)	(0.002)	(0.016)
Leverage	-0.000***	-0.000	-0.000***	-0.000
	(0.000)	(0.001)	(0.000)	(0.001)
Capital Exp.	-0.138***	-0.153***	-0.145***	-0.155***
	(0.004)	(0.006)	(0.004)	(0.006)
Lag Liquidity		0.238***		0.236***
		(0.012)		(0.013)
N	155133	99977	150468	96509

Robust standard errors are in brackets and are clustered at firm level.
* p<0.10, ** p<0.05, *** p<0.01

a frictional decentralized market, or DM; and a frictionless centralized market, or CM. This alternating-market structure is adapted from Lagos and Wright (2005). It is ideal for our purposes because in a stylized way the CM and DM correspond to primary and secondary capital markets. Moreover, it features an asynchronization of expenditures and receipts – reallocation occurs in the DM while profits accrue in the CM – crucial to any analysis of money or credit. It is also tractable and flexible.[5]

In the CM agents consume a numeraire good c, supply labor hours h, settle debt d, and adjust their portfolios of capital k and money m. In the DM they meet bilaterally and potentially retrade k, where gains from trade arise due to idiosyncratic productivity shocks. Agents discount between the CM and DM using $\beta \in (0, 1)$, but without loss of generality not between the DM and CM. Given time endowment 1, their utility over consumption and leisure is $U(c, 1-h) = u(c) + \xi(1-h)$, where $\xi > 0$ is a parameter, and $u'(c) > 0 > u''(c)$. As is well known, quasi-linear utility greatly enhances tractability in environments like this,

[5]By flexible we mean the environment accomodates many different specifications for search, price determination, etc. See Lagos et al. (2017) for a survey of papers using it in a variety of applications.

as indicated by the results in Lemma 1 and 2 below. However, following Wong (2016), one can show these results also hold for any monotone and concave $U(c, 1-h)$ such that $U_{11}U_{22} = U_{12}^2$, so one could use, say, $U = c^\gamma (1-h)^{1-\gamma}$ or $U = [c^\gamma + (1-h)^\gamma]^{1/\gamma}$.

Alternatively, as in Rocheteau et al. (2008) one can show that the results hold for any monotone and concave $U(c, 1-h)$ if labor is indivisible, $h \in \{0, 1\}$, and agents trade using employment lotteries as in Rogerson (1988). This is very relevant here when we compare our results on business cycles to those from other models, including the one in Hansen (1985). That is considered the standard model in the RBC (real business cycle) literature, as it jettisons many of the bells and whistles in Kydland and Prescott (1982) without a big impact on results, then improves on its performance by adding indivisible labor. A special case of our environment, with no idiosyncratic productivity shocks, is exactly Hansen's (1985) indivisible-labor model.[6]

We interpret agents as owning their own firms; we can let them own shares in many firms, but given Lemma 1 below there is no demand for that diversification. They have production functions satifying CRS (constant returns to scale)

$$f(k, h) = (A\varepsilon k)^{1-\eta} h^\eta,$$

where k is capital, h is labor, ε is an idiosyncratic productivity factor and A is aggregate productivity.[7] The firm-specific ε is drawn from a time-invariant distribution $F(\varepsilon)$. Aggregate uncertainty comes from A, which is constant for now; aggregate shocks come later. While the cross section of ε is constant, for a firm it can be persistent: using subscript $+$ to indicate next period, ε_+ is drawn from conditional distribution $Q(\cdot|\varepsilon)$, realized at the start of the DM after the CM closes. That generates gains from trade in the secondary capital market because ε is realized after firms choose k in the primary market, capturing the idea that it can be desirable to adjust k when new information arrives.

[6]One interpretation is that with indivisible labor and lotteries agents act *as if* $U(c, 1-h)$ were quasi-linear. Also it generates unemployment, not just leisure, but we do not dwell on that except in fn. 17. Note also that we consider Hansen (1985) the textbook model, as opposed to the monetary version in Cooley and Hansen (1989), because money is introduced there by a CIA constraint on household consumption while here it is used by firms for capital reallocation. Therefore, if our ε shocks are shut down, as a special case the formulation reduces to Hansen (1985) rather than Cooley and Hansen (1989).

[7]We assume CRS because it implies efficient DM trade involves full sales, and partial sales occur only due to liquidity problems. With DRS it is harder (but not impossible) to get full sales; it more likely to have partial sales that equate the marginal product of capital across the two firms.

In the CM, a firm with (k,ε) chooses labor demand $\tilde{h}$, distinguished from labor supply of its owner h, to maximize profit,

$$\Pi\left(k,\varepsilon\right)=\max_{\tilde{h}}\{\left(A\varepsilon k\right)^{1-\eta}\tilde{h}^{\eta}-w\tilde{h}\}.$$

(Note Π also depends on w, but that remains implicit to ease notation.) The solution is

$$\tilde{h}\left(k,\varepsilon\right)=\left(\frac{\eta}{w}\right)^{\frac{1}{1-\eta}}A\varepsilon k, \tag{1}$$

and therefore $\Pi\left(k,\varepsilon\right)=B\left(w\right)\varepsilon k$, where

$$B(w)\equiv\left(\frac{\eta}{w}\right)^{\frac{\eta}{1-\eta}}\left(1-\eta\right)A. \tag{2}$$

Due to CRS, profit is liner in εk, which means efficient bilateral reallocation entails full sales: the firm with higher ε should get all the capital.

Denote the CM and DM value functions by W and V. Then

$$W(\Omega,\varepsilon)=\max_{c,h,\hat{k},\hat{z}}\left\{u(c)-\xi h+\beta\mathbb{E}_{\hat{\varepsilon}|\varepsilon}V_{+}(\hat{k},\hat{z})\right\}, \tag{3}$$
$$\text{st } c=\Omega+(1-\tau_h)wh-\hat{z}\phi/\phi_{+}-\hat{k},$$

where Ω is wealth, k and z are capital and real money balances at the start of the CM while $\hat{k}$ and $\hat{z}$ are capital and real money balances at the end, τ_h is a labor income tax, and $\mathbb{E}_{\hat{\varepsilon}|\varepsilon}$ denotes the expectation wrt $\hat{\varepsilon}$ conditional on ε. Note that $z=\phi m$ is defined as money m times its price ϕ in terms of c, and the price of real balances next period $\hat{z}$ in terms of current c is the gross inflation rate $1+\pi=\phi/\phi_{+}$. Wealth is given by

$$\Omega=(1-\tau_k)B(w)\varepsilon k+(1-\delta)k+z-d-T,$$

where δ is the depreciation rate, τ_k is a capital income tax, T is a lump-sum tax (or transfer if $T<0$), and d is debt brought in from the previous DM.

Assume an interior solution for $h\in[0,1]$ and strict concavity of V_{+}, which can be verified later. Then, using the budget constraint to eliminate h, rewrite (3) as

$$\begin{aligned} W\left(\Omega,\varepsilon\right) &= \frac{\xi\Omega}{(1-\tau_h)w}+\max_{c}\left\{u(c)-\frac{\xi c}{(1-\tau_h)w}\right\} \\ &\quad+\max_{\hat{k},\hat{z}}\left\{-\frac{\xi\left(\hat{z}\phi/\phi_{+}+\hat{k}\right)}{(1-\tau_h)w}+\beta\mathbb{E}_{\hat{\varepsilon}|\varepsilon}V_{+}\left(\hat{k},\hat{m},\hat{\varepsilon}\right)\right\}. \end{aligned} \tag{4}$$

Conveniently, the portfolio problem – i.e., the choice of $(\hat{k}, \hat{z})$ – is isolated in the last term, and that immediately yields these results:

Lemma 1 *$W(\Omega, \varepsilon)$ is linear in Ω with slope $\xi/(1-\tau_h)w$.*

Lemma 2 *$(\hat{k}, \hat{z})$ solves*

$$-\frac{\xi}{(1-\tau_h)w} + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} \frac{\partial V_+(\hat{k}, \hat{z}, \hat{\varepsilon})}{\partial \hat{k}} \leq 0, \ = 0 \text{ if } \hat{k} > 0; \tag{5}$$

$$-\frac{\xi}{(1-\tau_h)w} \frac{\phi}{\phi_+} + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} \frac{\partial V_+(\hat{k}, \hat{z}, \hat{\varepsilon})}{\partial \hat{z}} \leq 0, \ = 0 \text{ if } \hat{z} > 0. \tag{6}$$

This means that $(\hat{k}, \hat{z})$ is the same for all agents with the same ε, although in general agents with different ε choose different $(\hat{k}, \hat{z})$.

To complete the CM problem, let $\hat{z}(\varepsilon)$ and $\hat{k}(\varepsilon)$ solve (5)-(6), which depend on ε because $\hat{\varepsilon}$ does, and note from (4) that c solves $u'(c) = \xi/(1-\tau_h)w$. Then the budget gives labor supply,

$$h(\Omega, \varepsilon) = \frac{c + \hat{k}(\varepsilon) + \hat{z}(\varepsilon)\phi/\phi_+ - \Omega}{(1-\tau_h)w}, \tag{7}$$

where dependence on w is implicit. Also, if Γ is the distribution of (k, z, ε) at the start of a period, its law of motion is

$$\Gamma_+(k, z, \varepsilon) = \int\limits_{\hat{k}(x) \leq k, \hat{z}(x) \leq z} Q(\varepsilon|x)\, dF(x). \tag{8}$$

Without aggregate shocks, while agents move around in the distribution the cross section is constant, but of course that is not true with aggregate shocks. In any case, there is a distribution across agents leaving the CM, but tractability is preserved since $\hat{k}$ and $\hat{z}$ depend only on ε (and not on past DM trades, as in models like Molico 2006 or Molico and Zhang 2006).

Now consider the DM, where with probability α each firm (owner) is randomly matched to a potential trading partner.[8] In a pairwise meeting the state variables of the parties are

[8] It may be interesting to consider competitive search, with directed rather than random matching and price posting rather than bargaining, but it would be complicated by having to solve for a cutoff, say ε^*, where firms above (below) ε^* act as buyers (sellers). Having said that, random matching is arguably nice if α is interpreted as the probability of meeting an *appropriate* counterparty – e.g., you meet someone for

$\mathrm{s} = (k, z, \varepsilon)$ and $\tilde{\mathrm{s}} = (\tilde{k}, \tilde{z}, \tilde{\varepsilon})$. Given $\varepsilon > \tilde{\varepsilon}$, there are gains from trade where the ε firm, called *the buyer*, gets some quantity $q(\mathrm{s}, \tilde{\mathrm{s}})$ of capital from the $\tilde{\varepsilon}$ firm, called *the seller*. Let $p(\mathrm{s}, \tilde{\mathrm{s}})$ be the value of any cash payment the buyer makes, and $d(\mathrm{s}, \tilde{\mathrm{s}})$ the value of any debt payment, i.e. promised payment deferred to the next CM (both cash and credit are available, but limited, as discussed below). Then

$$V(k, z, \varepsilon) = W(\Omega, \varepsilon) + \alpha \int\limits_{\varepsilon > \tilde{\varepsilon}} S^b(\mathrm{s}, \tilde{\mathrm{s}}) d\Gamma(\tilde{\mathrm{s}}) + \alpha \int\limits_{\varepsilon < \tilde{\varepsilon}} S^s(\tilde{\mathrm{s}}, \mathrm{s}) d\Gamma(\tilde{\mathrm{s}}), \tag{9}$$

where $S^b(\cdot)$ and $S^s(\cdot)$ are buyer and seller surpluses, which by Lemma 1 are

$$S^b(\mathrm{s}, \tilde{\mathrm{s}}) = \frac{\xi \{[(1 - \tau_k) \varepsilon B(w) + 1 - \delta] q(\mathrm{s}, \tilde{\mathrm{s}}) - p(\mathrm{s}, \tilde{\mathrm{s}}) - d(\mathrm{s}, \tilde{\mathrm{s}})\}}{w(1 - \tau_h)}; \tag{10}$$

$$S^s(\tilde{\mathrm{s}}, \mathrm{s}) = \frac{\xi \{p(\tilde{\mathrm{s}}, \mathrm{s}) + d(\tilde{\mathrm{s}}, \mathrm{s}) - [(1 - \tau_k) \tilde{\varepsilon} B(w) + 1 - \delta] q(\tilde{\mathrm{s}}, \mathrm{s})\}}{w(1 - \tau_h)}. \tag{11}$$

We distinguish between two types of reallocation: a full sale, $q(\mathrm{s}, \tilde{\mathrm{s}}) = \tilde{k}$, which might be described as one firm taking over the other; and a partial sale, $q(\mathrm{s}, \tilde{\mathrm{s}}) \in (0, \tilde{k})$, which leaves both operating in the next CM. While full sales are socially efficient, they may not happen if buyers are limited in ability to pay. Up-front cash payments are obviously constrained by feasibility, $p \leq z$, while deferred payments are constrained by $d \leq D$, where in general the debt limit is

$$\bar{D} = \chi_0 + \chi_\Pi \Pi + \chi_k (1 - \delta) k + \chi_q (1 - \delta) q.$$

The first term represents unsecured debt, which can be taken as a parameter, or can be endogenized, as in Kehoe and Levine (1993). The second represents debt secured by future profit, as in Holmstrom and Tirole (1998). The third and fourth represent debt secured by a existing and by newly-acquired capital, as in Kiyoaki and Moore (1997).

As is standard, we call χ_Π, χ_k and χ_q pledgeability parameters and impose $\chi_j \in [0, 1]$. The usual story for $\chi_j < 1$ is that only some of your assets can be pledged as collateral, since if you renege on a payment only a fraction χ_j can be seized, while you abscond

sure, but only with probability α do they have the kind of k you want or want the kind you have, capturing *specificity* notion mentioned in fn. 1. This is invoked in other applications of search, like labor, where you might think you can always find a firm but they may not be hiring in your area. Our model also has a matching friction in that a seller (buyer) of k would like to meet someone with the highest (lowest) ε, but we assume they meet at random; to see how it matters, one can shut off that friction (see in Section 6).

with the rest. An alternative approach based on private information is provided by Li et al. (2011): holding more assets than can be used to secure debt signals that they are high quality. In any case, we usually impose $\chi_\Pi = \chi_k = 0$ below to focus on the others.

The next step is to determine the terms of trade, $q(\mathbf{s}, \tilde{\mathbf{s}})$, $p(\mathbf{s}, \tilde{\mathbf{s}})$ and $d(\mathbf{s}, \tilde{\mathbf{s}})$. We use Kalai's (1977) bargaining solution with θ denoting buyers' share, which can be found by solving

$$\max_{d,p,q} S^b(\mathbf{s}, \tilde{\mathbf{s}}) \text{ st } (1-\theta) S^b(\mathbf{s}, \tilde{\mathbf{s}}) = \theta S^s(\tilde{\mathbf{s}}, \mathbf{s}), \tag{12}$$

with our additional constraints $q \le \tilde{k}$, $p \le z$ and $d \le (1-\delta)(\chi_q q + \chi_k k)$.[9]

Proposition 1 *In any DM meeting $(\mathbf{s}, \tilde{\mathbf{s}})$ with $\varepsilon > \tilde{\varepsilon}$, there is threshold*

$$\bar{\varepsilon} = \bar{\varepsilon}(\mathbf{s}, \tilde{\mathbf{s}}) \equiv \frac{[z + \chi_k(1-\delta)k]/\tilde{k} - (1-\delta)\left(1-\chi_q\right)}{(1-\tau_k)(1-\theta)B(w)} - \frac{\theta\tilde{\varepsilon}}{1-\theta}, \tag{13}$$

with the following properties: (i) If $\varepsilon > \bar{\varepsilon}_b$ there is a partial sale $q = Q < \tilde{k}$, where

$$Q = Q(\mathbf{s}, \tilde{\mathbf{s}}) \equiv \frac{z + (1-\delta)\chi_k k}{(1-\tau_k)B(w)[(1-\theta)\varepsilon + \theta\tilde{\varepsilon}] + (1-\delta)\left(1-\chi_q\right)}, \tag{14}$$

and payments satisfy the constraints at equality, $p = z$ and $d = (1-\delta)(\chi_q Q + \chi_k k)$. (ii) If $\varepsilon < \bar{\varepsilon}_b$ there is a full sale $q = \tilde{k}$, and the total payment is

$$p + d = \{(1-\tau_k)B(w)[(1-\theta)\varepsilon + \theta\tilde{\varepsilon}] + 1 - \delta\}\tilde{k},$$

but the mix between p and d is irrelevant as long as $p \le z$ and $d \le (1-\delta)(\chi_q \tilde{k} + \chi_k k)$.

All proofs are in Appendix B. The above results, illustrated in Figure 4, accord well with intuition: if a buyer is flush with liquidity there is a full sale and we say *the seller stocks out*; but if liquidity is tight there is a partial sale and we say *the buyer cashes out*. Inefficient partial sales occur when the unconstrained price (i.e., the one that would prevail with perfect credit, $\chi_0 = \infty$) is high, which as Figure 4 shows happens when ε and $\tilde{\varepsilon}$ or big. Therefore, sometimes meetings with lucrative potential deals, in the sense that ε is very big, yield only partial sales as the buyer cashes out.

[9]The Kalai solution emerges as the outcome of his axioms, but similar to generalized Nash, it can be characterized by a simple maximization problem. Here Kalai and generalized Nash are the same if liquidity constraints are slack, but not if they bind. Since Aruoba et al. (2007), Kalai bargaining is known to have several advantages in models of money or credit and hence has become popular in this literature.

The last step before defining equilibrium is to discuss market clearing in the CM, where m, c and h are traded. By Walras' law one can be ignored, and we choose h; indeed, h does not appear in the definition below, but can always be recovered using (7). For m, let the aggregate supply M grow at rate $\mu > \beta - 1$, with changes engineered in the CM as follows: add the seigniorage to revenue from τ_h and τ_k, subtract government spending G, and set the lump sum T to balance the public budget each period. Then money and goods market clearing are given by

$$\phi M_+ = \int \hat{z}(\varepsilon)\, dF(\varepsilon) \text{ and } c + K_+ + G = Y + (1-\delta) K, \tag{15}$$

where $Y = B(w)\bar{K}/(1-\eta)$ is total output, $K_+ = \int \hat{k}(\varepsilon)\, dF(\varepsilon)$ is gross investment, and $\bar{K}$ is effective capital weighted by productivity after DM trade,

$$\begin{aligned} \bar{K} &= \alpha \iint_{\varepsilon > \tilde{\varepsilon}} \varepsilon \left[k + q(\mathbf{s}, \tilde{\mathbf{s}})\right] d\Gamma(\tilde{\mathbf{s}}) d\Gamma(\mathbf{s}) \\ &\quad + \alpha \iint_{\varepsilon < \tilde{\varepsilon}} \varepsilon \left[k - q(\tilde{\mathbf{s}}, \mathbf{s})\right] d\Gamma(\tilde{\mathbf{s}}) d\Gamma(\mathbf{s}) + (1-\alpha) \int \varepsilon k d\Gamma(\mathbf{s}). \end{aligned} \tag{16}$$

This leads to the following definition, where when we say that paths are bounded it means the usual transversality conditions in these kinds of models, $\beta^t u'(x_t)\hat{k}_t \to 0$ and $\beta^t u'(x_t)\hat{z}_t \to 0$ (e.g., see Rocheteau and Wright 2013):

Definition 1 *Given initial conditions for (z, k) and paths for (M, G, τ_h, τ_k) equilibrium is a list of nonnegative, bounded paths for $(c, \hat{z}, \hat{k}, q, p, d, \phi, w, \Gamma)$, where $\hat{z} = \hat{z}(\varepsilon)$ and $\hat{k} = \hat{k}(\varepsilon)$ for each agent while $q = q(\varepsilon, \tilde{\varepsilon})$, $p = p(\varepsilon, \tilde{\varepsilon})$ and $d = d(\varepsilon, \tilde{\varepsilon})$ for each pair, satisfying at every date: (i) in the CM $(c, \hat{z}, \hat{k})$ solves (3); (ii) in the DM (q, p, d) are given by Proposition 1; (iii) markets clear as defined by (15); and (iv) Γ evolves according to (8). It is a monetary equilibrium if $\hat{z} > 0$.*

To discuss steady state, let G, τ_h, τ_k and μ be constant. If $\mu \neq 0$ then ϕ generally changes over time, but $\phi M = z$ does not if $\phi/\phi_+ = 1 + \mu$, so we eliminate ϕ from the list of variables in the following definition:

Definition 2 *Steady state is a time-invariant list $(c, z, k, q, p, d, w, \Gamma)$ satisfying Definition 1 except for the initial conditions. It is a monetary steady state if $z > 0$.*

As a digression, note that while monetary policy is specified above in terms of μ, we can instead target inflation π or an illiquid nominal rate ι, and in steady state these are equivalent: pegging μ is the same as pegging $\pi = \mu$ or $\iota = (1+\pi)/\beta - 1$. To be precise, define illiquid interest rates as follows: $1+r$ is the amount of c agents require in the next CM to give up 1 unit in this CM; and $1+\iota$ is similar except m replaces c. If there is another liquid asset, as in Appendix C, it is also equivalent to target its nominal rate, and over some range it is possible to target the real rates on certain assets but not others. In any case, in what follows we impose $\iota > 0$, but consider the limit as $\iota \to 0$, which is what Monetarists call the Friedman rule and Keynesians call the zero lower bound.

Continuing towards a characterization equilibrium, the next step is to derive the marginal value of capital entering the DM,

$$\begin{aligned}\frac{\partial V}{\partial k} = \frac{\xi}{(1-\tau_h)w} \Bigg\{ (1-\tau_k)B(w) \Bigg[& \varepsilon + \alpha(1-\theta) \int_{\mathcal{S}_s(\mathbf{s})} (\tilde{\varepsilon} - \varepsilon)\, d\Gamma(\tilde{\mathbf{s}}) \\ & + \alpha\theta(1-\delta)\chi_k \int_{\mathcal{S}_b(\mathbf{s})} \frac{\varepsilon - \tilde{\varepsilon}}{\Delta(\varepsilon,\tilde{\varepsilon})} d\Gamma(\tilde{\mathbf{s}}) \Bigg] + 1 - \delta \Bigg\},\end{aligned} \tag{17}$$

where to save space $\Delta(\varepsilon, \tilde{\varepsilon})$ denotes the denominator in (14), while

$$\mathcal{S}_s(\mathbf{s}) = \{\tilde{\mathbf{s}} : \tilde{\varepsilon} > \varepsilon,\ \tilde{\varepsilon} < \overline{\varepsilon}(\tilde{\mathbf{s}}, \mathbf{s})\} \text{ and } \mathcal{S}_b(\mathbf{s}) = \{\tilde{\mathbf{s}} : \tilde{\varepsilon} < \varepsilon,\ \varepsilon > \overline{\varepsilon}(\mathbf{s}, \tilde{\mathbf{s}})\} \tag{18}$$

are the set of meetings where you sell to buyers and they are constrained, and the set of meetings where you buy from sellers and you are constrained, respectively. In words, a marginal unit of k has four potential benefits: (i) You can get its contribution to CM production, the first term in square brackets, which is just ε because $(1-\tau_k)B(w)$ outside the brackets converts εk into disposable income. (ii) You can get its value from a DM sale, the second term in brackets, since you sell all of your k when you meet someone with $\tilde{\mathbf{s}} \in \mathcal{S}_s(\mathbf{s})$ and enjoy a share $1-\theta$ of the total surplus. (iii) You can get its DM collateral value, captured by the third term in brackets, since you hit your liquidity constraint when you buy from someone with $\tilde{\mathbf{s}} \in \mathcal{S}_b(\mathbf{s})$ and enjoy a share θ of the surplus. (iv) You can get the resale value of $1-\delta$ in the last term.

Similarly, derive

$$\frac{\partial V}{\partial z}=\frac{\xi}{(1-\tau_h)w}\left[1+(1-\tau_k)B\left(w\right)\alpha\theta\int\limits_{\mathcal{S}_b(\mathbf{s})}\frac{\varepsilon-\tilde{\varepsilon}}{\Delta\left(\varepsilon,\tilde{\varepsilon}\right)}d\Gamma\left(\tilde{\mathbf{s}}\right)\right]. \tag{19}$$

In words, a marginal unit of z has two potential benefits: (i) You can get its CM purchasing power. (ii) You can get its DM purchasing power, since you hit your liquidity constraint as a buyer when you meet someone with $\tilde{\mathrm{s}} \in \mathcal{S}_b\left(\mathrm{s}\right)$ and you enjoy a share θ of the surplus.

Now combine (17)-(19) with the FOCs in Lemma 2 at equality to get

$$\frac{1}{w}=\frac{\beta(1-\tau_k)B\left(w_+\right)}{w_+}\mathbb{E}_{\varepsilon_+|\varepsilon}\left[\varepsilon_++\alpha\left(1-\theta\right)I_s+\alpha\theta\left(1-\delta\right)\chi_k I_b\right]+\frac{\beta\left(1-\delta\right)}{w_+} \tag{20}$$

$$\frac{Z}{w}=\frac{\beta Z_+}{w_+\left(1+\mu\right)}\mathbb{E}_{\varepsilon_+|\varepsilon}\left[1+(1-\tau_k)B\left(w_+\right)\alpha\theta I_b\right], \tag{21}$$

where to save space we define

$$I_s\equiv\int\limits_{\mathcal{S}_s(\mathbf{s}_+)}\left(\tilde{\varepsilon}_+-\varepsilon_+\right)d\Gamma_+\left(\tilde{\mathbf{s}}_+\right)\text{ and }I_b\equiv\int\limits_{\mathcal{S}_b(\mathbf{s}_+)}\frac{\varepsilon_+-\tilde{\varepsilon}_+}{\Delta\left(\varepsilon_+,\tilde{\varepsilon}_+\right)}d\Gamma\left(\tilde{\mathbf{s}}_+\right).$$

These Euler equations play a big role in what follows.

Before presenting more substantive results, we need to say how banks enter the picture. So far this was not mentioned, as it seemed best to first present the framework without banking, but we want it for the quantitative work below. To begin, suppose after the CM closes and before the DM opens some information is revealed that affects agents' desired liquidity. Then as in Berentsen et al. (2007), banks can help reallocate $\hat{z}$. Now, we want the information to be minimal, because different signals generally entail different demands for $\hat{z}$, making it harder to solve the model. Therefore it is assumed that the only information revealed is whether an agent will have a meeting in the upcoming DM. This signal entails just two types: those that will not have a DM meeting, who hold excess cash; and those that will have a meeting, who could use more.

This liquidity mismatch problem – actually, liquidity mismatch opportunity – introduces a role for banks similar to the one in models based on Diamond and Dybvig (1982).[10]

[10] As one way to envision this, imagine agents making trips to the bank between the CM and DM. However, we do not want them to trade $\hat{k}$ at the bank, that is supposed to happen in the DM; therefore, to that end we invoke "sequential service" as discussed in the context of Diamond-Dybvig by Wallace (1988). Then agents holding $\hat{k}$ never meet each other, so they cannot physically trade $\hat{k}$ with each other, but bankers can still issue (lend) them liquid claims to each others' deposits.

However, different from most of those models, and similar to Berentsen et al. (2007), our banks take deposits and make loans in money, not goods. Also, different from many of those models, an infinite horizon is crucial both for having money valued in the first place and for incentives based on reputation. In particular, it is hard for agents to trade $\hat{z}$ among themselves using promises of repayment in the next CM, for the same reason it is hard to trade $\hat{k}$ using such promises: lack of commitment and lack of concern for reputation. It is reasonable to say bankers have more concern for reputation, as they are not anonymous, so their promises to honor deposits are relatively credible. Also, they can have comparative advantage in collecting debts. In this scenario, bankers serve as intermediaries facilitating exchange of $\hat{z}$ across agents.[11]

For our purposes, the important implication is that this props up money demand. The insight in Berentsen et al. (2007) is that banking makes it less costly to hold cash, because if you find yourself with more than you need you can keep it in the bank, at interest financed by loans to those who want more. Thus banking reduces the cost of getting stuck with idle cash balances. Moreover, the only change in the equilibrium conditions is that α no longer appears in (21). To see the equations explicitly one can look at, e.g., He et al. (2015), a different bit related application; here we think the intuition should suffice: by lending $\hat{z}$ when you will have no meeting in the upcoming DM, you get the same marginal benefit as those that will have a meeting. Thus α drops out of the Euler equation for $\hat{z}$, and that means monetary equilibrium exists for a bigger range of parameters. Of course we could just set $\alpha = 1$ and similarly prop up money demand, but that has other implications, e.g., for DM trading volume, and hence is not equivalent. In fact, in the calibration below α is far less than 1, as is necessary to get the right amount of partial sales.

4 Analytic Results

To develop results and insights, suppose for now that ε is i.i.d., which simplifies the analysis because then $\mathbb{E}_{\hat{\varepsilon}|\varepsilon} V_+(\hat{k}, \hat{z})$ is independent of ε, so everyone has the same $(\hat{k}, \hat{z}) =$

[11] We think this captures in spirit the role of banks in both Gertler and Kiyotaki (2015) and Gu et al. (2013), even if the fomer are more concerned with macro applications and the latter with microfoundations. Now fleshing out the details of how banking works and deriving more implications may be interesting, but would take us too far afield for this project.

(K_+, Z_+) leaving the CM. To see what this implies, first, define

$$L \equiv \frac{(Z+\chi_0)/K - (1-\delta)\left(1-\chi_q-\chi_k\right)}{(1-\tau_k)B(w)}, \tag{22}$$

a normalized notion of liquidity determining when the constraint binds, and note we now include χ_0 (unsecured credit). Using L and abusing notation slightly, write

$$\mathcal{S}_b(L) \equiv \left\{(\varepsilon,\tilde{\varepsilon}) : \varepsilon > \tilde{\varepsilon},\, \varepsilon > \frac{L-\theta\tilde{\varepsilon}}{1-\theta}\right\} \text{ and } \mathcal{S}_s(L) \equiv \left\{(\varepsilon,\tilde{\varepsilon}) : \varepsilon > \tilde{\varepsilon},\, \tilde{\varepsilon} < \frac{L-\theta\varepsilon}{1-\theta}\right\}$$

for the sets of meetings where partial and full sales occur, as defined in (18), except here they are functions of L. Also use L to write the effective capital stock defined in (16) as $\bar{K} = J(L,w)K$, with

$$J(L,w) \equiv \mathbb{E}\varepsilon + \alpha I_s(L) + \alpha\left[(1-\tau_k)B(w)L + (1-\delta)\left(1-\chi_q\right)\right] I_b(L),$$

where, slightly abusing notation again,

$$I_b(L) = \iint\limits_{\mathcal{S}_b(L)} \frac{\varepsilon-\tilde{\varepsilon}}{\Delta(\varepsilon,\tilde{\varepsilon})} dF(\varepsilon)dF(\tilde{\varepsilon}) \text{ and } I_s(L) = \iint\limits_{\mathcal{S}_s(L)} (\tilde{\varepsilon}-\varepsilon)\, dF(\varepsilon)dF(\tilde{\varepsilon}).$$

Given this, in steady state the Euler equations become

$$r+\delta = \left[\mathbb{E}\varepsilon + \alpha(1-\theta) I_s(L) + (1-\delta)\chi_k\iota\right] B(w)(1-\tau_k) \tag{23}$$

$$\iota = \alpha\theta \iint\limits_{\mathcal{S}_b(L)} \frac{(\varepsilon-\tilde{\varepsilon})\, dF(\tilde{\varepsilon})dF(\varepsilon)}{(1-\theta)\varepsilon + \theta\tilde{\varepsilon} + \frac{(1-\delta)\left(1-\chi_q\right)}{(1-\tau_k)B(w)}}, \tag{24}$$

where r and ι are the illiquid real and nominal rates defined above. Also, goods market clearing is

$$u'^{-1}\left[\frac{\xi}{(1-\tau_h)\,w}\right] + G = \left[\frac{B(w)\,J(L,w)}{1-\eta} - \delta\right] K. \tag{25}$$

Now (23)-(25) constitute three equations in (K, Z, w), while if ε were not i.i.d., K and Z would be functions, not just numbers.

For what it's worth, (23) and (24) can be called the IS and LM curves, representing the demand for Investment and Liquidity. Both depend on credit conditions as captured by the χ's, fiscal policy as captured by G and the τ's, and monetary policy as captured by ι or π or μ. While the framework is evidently different from the IS-LM model many people teach in undergrad macro, it can be used the same way by shifting curves (see below). The only complication is that wages here are endogenous, but in principle one can solve (25) for w and insert it into (23)-(24) to get two equations in (K, Z).[12]

In practice, it is better to regard (23)-(24) as two equations in (L, B), independent of other variables, and then, given (L, B), get Z/K and w from (12) and (22), and finally get K from (25). In Figure 5, an intersection of (23)-(24) in (L, B) space is a monetary steady state. Appendix B proves this:

Proposition 2 *If θ is not too small, while χ_0 and χ_k are not too big, there exists a unique monetary steady state if and only if $\iota < \bar{\iota}$, where $\bar{\iota} > 0$.*

These are sharp results even if they depend on parameters. We say this because it is well known from related work that under Kalai bargaining existence of monetary equilibria requires θ above and ι below thresholds, and is obvious that it also requires χ_0 and χ_k below thresholds since if credit is too easy money is irrelevant. We do not need restrictions on χ_q, however, other than the maintained $\chi_q \leq 1$. The reason is that even if $\chi_q = 1$, credit secured only by $(1 - \delta) q$ cannot even cover sellers' outside option, since they get $(1 - \delta) q$ before even producing. Thinking of credit secured by q like a mortgage, where the collateral is the house being purchased, we interpret this as saying that even at $\chi_q = 1$ buying used capital requires either a cash down payment or another line of credit.

Table 5 shows the effects of policy and credit conditions on (K, Z, w), output Y and Φ, with Φ defined as the probability of a full sale conditional on a meeting. Here $\pm$ means the result can go either way (and not that we failed to figure it out). In particular, the nonmonotonicity of K and Y with respect to ι is confirmed in the numerical work. A perhaps surprising result is the ambiguous impact of ι on Z, although $\partial Z/\partial\iota < 0$ holds in the numerical work. Notice $\partial Z/\partial\chi_0 < 0$ is unambiguous: related to the discussion on credit conditions and money demand, higher χ_0 reduces the need for cash, putting upward pressure on the price level and inflation in the short run, even if it is pinned down by money growth $\phi/\phi_+ = 1 + \mu$, by definition of stationary equilibrium. Also related to

[12]One trick is to assume $f(k, h) = h + \tilde{f}(k)$ to pin down $w = 1$; however, that it is obviously an extreme specification. Another is to set $\theta = 1$ so that (23) pins down $B(w)$, thus determining w endogenously but independently of other variables; however, that is also extreme, in the sense that varying θ has interesting positive and normative implications we do not want to miss.

that discussion the effects on Φ are unambiguous: in the long run the fraction of full sales goes down with ι and up with χ_k or χ_q.

Table 5: Comparative Statics

	ι	χ_0	χ_k	χ_q	τ_k	τ_h
K	$\pm$	0	$+$	$-^*$	$-$	$-$
Z	$\pm$	$-$	$\pm$	$\pm$	$\pm$	$-$
w	$+^*$	0	$+$	$+$	$-$	0
Y	$\pm$	0	$+$	$-^*$	$-$	$-$
Φ	$-$	0	$+$	$+$	$+$	0

Note: Results assume χ_q is big, and $*$ requires θ big.

These effects are illustrated by shifting IS and LM curves in Figure 5, and we make much use of that in proving the results in Appendix B. Consider for illustration $\chi_k = 0$, so that changes in ι shift LM but not IS. Intuitively, higher ι increases the cost of liquidity, so agents only want to hold the same Z if the benefit to reallocation is higher, which means B must rise. As shown, LM shifts to the northwest leading to lower L, which implies fewer full sales, more partial sales, and less total reallocation, plus higher B, which implies higher profit per unit of capital. By continuity this is still valid if $\chi_k > 0$ is not too big, but if χ_k is big, while L is still decreasing in ι, B can be increasing or decreasing. Having $\partial k/\partial \iota > 0$ is similar to the Mundell-Tobin effect in the superficial sense that higher ι makes agents want to shift out of Z and into K, but the reason here is that Z and K are substitutes as payment instruments when $\chi_k > 0$.

In terms of efficiency, the Appendix solves the planner problem and compares it to equilibrium with no fiscal distortions, $\tau_k = \tau_h = 0$. For efficient q, conditional on other variables, we need full sales in all meetings and hence $\iota \to 0$. For efficient K we need sellers in the DM to reap the full benefit of their investments, which requires $\theta = 0$, but $\theta = 0$ implies monetary equilibrium cannot exist, and more generally θ low implies there is underinvestment in Z. This is a two-sided holdup problem: given $\iota > 0$ we need θ big to support efficient money demand, and θ small to support efficient capital demand; no $\theta \in [0, 1]$ delivers both. However, Appendix B proves that for any θ steady state is efficient if there is no monetary or labor wedge, $\iota \to 0$ and $\tau_h = 0$, and we implement a corrective subsidy on capital formation:

Proposition 3 *Efficiency is not possible at $\iota > 0$. When $\iota \rightarrow 0$ monetary steady state is efficient if $\tau_h = 0$ and $\tau_k = \tau_k^*$, where $\tau_k^* \leq 0$ with strict inequality unless $\theta = 0$, is given by*

$$\tau_k^* = 1 - \frac{r + \delta}{(1-\eta) A \left[\frac{\eta u'(c^*)}{\xi}\right]^{\frac{\eta}{1-\eta}} \left[\int_0^\infty \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha (1-\theta) \int_{\hat{\varepsilon}<\tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon}) dF(\tilde{\varepsilon}) dF(\hat{\varepsilon})\right]}, \quad (26)$$

and c^* is consumption at the planner's solution.

While formula (26) may appear imposing, we will put numbers on it below.

Before moving to the quantitative work let us consider a pure-credit setup, pure in two senses: (i) $\phi M = 0$ (no money); and (ii) $\chi_0 > 0$, $\chi_j = 0$ for $j \neq 0$ (no collateral). Of course pure credit does not mean perfect credit unless χ_0 is big, since now DM trade is constrained by $d \leq \chi_0$. The capital equation (23) and market clearing (25) are similar to above, but now they immediately yield two equations in (K, w). In terms of efficiency, $\iota \rightarrow 0$ in the monetary version is equivalent to having χ_0 big enough to render $d \leq \chi_0$ slack. Thus we have the following:[13]

Proposition 4 *With pure credit, steady state exists and is unique if α is not too big. It is efficient if χ_0 is sufficiently large, $\tau_h = 0$ and $\tau_k = \tau_k^*$ as given in (26).*

5 Quantitative Results

As many of our parameters are standard in the RBC literature, we follow that approach where possible (e.g., see Gomme and Rupert 2007). In particular, while in general all the parameters are set simultaneously to hit all the targets, it is useful to discuss how each parameter is set to match one observation when that observation has the most obvious or important impact. Another preliminary remark is that ε is i.i.d. for now, but that is relaxed in Section 6. Also, the sample period is 1984 to 2018 (for reasons given in in fn. 3).

5.1 Calibration

The calibrated parameters are in Table 6; here is how we got them. Average annual (CPI) inflation π over the period is 2.68%. For the average illiquid nominal rate, the AAA corpo-

[13]One reason to present these result is that most papers on capital reallocation do not have money. While we consider that a deficiency, a nonmonetary version of our framework faciliates comparison with that work. One can also study nonmonetary versions with collateralized credit, as discussed in Section 6.

rate bond nominal yield ι over the period is 6.72%. We considered options, including the T-bill yield, but settled on this for three reasons: (i) We are, after all, studying corporations. (ii) Yields on corporate bonds are somewhat high, presumably in part because they are somewhat risky, but our agents are effectively risk neutral on that dimension by Lemma 1, so the measure corresponds well to our definition of ι (the money agents require in the next CM to give up a dollar in this CM). (iii) It is generally accepted in finance that corporate bonds are less liquid than T-bills, even if this is sometimes described as less "convenient" (e.g., Krishnamurthy and Vissing-Jorgensen 2012). From π and ι we get the average real illiquid rate $1+r=(1+i)/(1+r)=1.0393$ and $\beta=1/(1+r)=0.962$.

We made an effort to be careful with the above numbers, as they are central to our approach, and are confident in this β even if the results are similar in a reasonable range around it. For other preference parameters, we use $u(c)=\log(c)$ and set the coefficient on leisure to $\xi=2.38$ so hours worked as a fraction of discretionary time are 33%, as reliably reported in time-use surveys for the representative US household, even if the key results are not sensitive to this. For technology we normalize $A=1$ and set labor's share to $\eta=0.60$, which is well within the range people use, although perhaps in the lower part of the range, which we feel is appropriate given the kinds of businesses we are trying to model. That is partially offset by having depreciation somewhat high, $\delta=0.125$, to yield a reasonable $K/Y=1.8$; but we also report results for $\delta=0.1$ (see below). Then G is 20% of steady state Y, while $\tau_k=0.25$ and $\tau_h=0.22$, reasonable approximations to US fiscal policy (Gomme and Rupert 2007).

Other parameters are less standard in textbook RBC theory but can still be set with discipline. The idiosyncratic productivity shock distribution is log-normal, $\log\varepsilon\sim N(\log\bar{\varepsilon},\sigma^2)$, with the mean $\bar{\varepsilon}$ set to normalize $\mathbb{E}\varepsilon=1$ and $\sigma=1.15$, in line with previous studies.[14] The DM arrival rate is set to $\alpha=0.118$ to match an R share of 30%, as reported in Section 2, and it should be clear how this is the right parameter for that target. For credit frictions, we set $\chi_q=0.836$ to match a P share of 31%, as reported in Section 2, and $\chi_k=0.0652$ to match firm money holdings of 5%, and it is clear how these parameters, describing credit limits, are natural for targeting the fraction of partial sales and the amount of cash firms

[14]A usual convention is to fit an AR(1) process for idiosyncratic productivity shocks, with an estimated persistence parameter 0.7 standard deviation of 0.25 for COMPUSTAT data.

Table 6: Calibrated Parameters and Targets

	Value	Explanation/Target		Value	Explanation/Target
ι	0.067	inflation rate π	α	0.118	R share 30% (35%)
β	0.962	illiquid real rate r	χ_q	0.836 (0.860)	P share 31%
ξ	2.38	labor hours h	χ_k	0.065 (0.057)	cash/output 5%
η	0.60	labor's share	τ_k	0.25	capital tax rate
δ	0.125 (0.10)	depreciation rate	τ_h	0.22	labor tax rate
$\bar{\varepsilon}$	-0.66	normalization	G	0.11	gov't 20% of Y
σ	1.15	COMPUSTAT	θ	0.50	symmetry

Note: numbers in parentheses are for the calibration with δ = 0.1.

hold in response to their credit limits.[15]

The remaining parameter is buyers' bargaining power, which we simply set to $1/2$, for several reasons. First, this means symmetric bargaining, which seems natural with ex ante identical agents. Second, this is conservative in the sense that we could only do better by choosing θ to match something. Third, when we tried freeing up θ it came out as almost exactly $1/2$. Finally, in what follows a lot of time and effort is spent discussing how results vary with θ, as we want to know where it matters, and by how much.

As mentioned, instead of $\delta = 0.125$ we also try $\delta = 0.1$, and recalibrating the other parameters, but they do not change much: as shown Table 6 by the numbers in parentheses, the only ones that move (up to three digits) are χ_q and χ_k. The calibration with $\delta = 0.1$ raises K/Y to 2.08, somewhat better than with $\delta = 0.125$, but cannot deliver an R share of 30%, so we end up with 35%. Hence there is a tradeoff between having δ a bit big and hitting the R share exactly, or having δ and K/Y slightly better but overshooting the R share. An argument for the former is that a big deal was made of the R share in Section 2, so it better be right here. A counter argument is based on the conventional wisdom that the data underestimate the R share. (Recall that they neglect small firms and those that are not publicly traded, neglect mergers, and include purchases but not rentals.) While 30% may well be an underestimate, it is hard to say if 35% is too low, too

[15] The number for firms' cash holdings is from FRED, Nonfinancial Corporate Business; Checkable Deposits and Currency, as befitting our not-so-narrow notion of money. We divide nominal GNP, as is standard in the money demand literature to make the series stationary, and get 5%. Note that we did not target the amount of cash used in reallocation, which as mentioned above is 42% by value (Thomson Reuters M&A Database, 1971-2018), but end up matching that very well.

high or just right. Therefore we report results for both, using as a benchmark the one with an R share of 30%, and indicating where things change with 35%.

In summary, in matching model and data we cannot get everything exactly right, but we are really close. Moreover, the bar was set high – e.g., most studies of reallocation make no attempt at matching money demand and vice versa. Our conclusion is that the model does a good job hitting the targets, which, it is worth emphasizing, are all first moments except the standard deviation of ε, and which are for the most part well measured.

5.2 Long-Run Results

We start with the impact of inflation on steady state. While Table 5 has analytic results on that, some can go either way, and, moreover, we want the magnitude, not just the sign. This is shown in Figure 6, where the x-axis is labeled ι since, in steady state, $1 + \iota = (1 + \pi)/\beta$ means eiter π or ι measure inflation. This uses the calibration with $\delta = 0.125$, but the picture is very similar with $\delta = 0.1$. Results are shown for three values of θ to assess the impact of bargaining power: our baseline $\theta = 1/2$, plus two higher values $2/3$ and $3/4$. One can of course choose lower θ, and for some issues that is interesting, but for most it not. One can actually see from the graphs what happens as θ falls below $1/2$ – simply shift the curves the other way – but note that low θ imply monetary equilibria exist only for very low ι, if they exist at all. The vertical lines show $\bar{\iota}$ in Proposition 2 above which monetary equilibria vanish: for $\theta = 0.5$, $\bar{\iota}$ is 12%; for for $\theta = 2/3$ it is 18%; and for $\theta = 3/4$ it is 22%.[16]

The top row the Figure shows standard macro variables. They are normalized to 100 at $\alpha = 0$, which can be interpreted as their values in the textbook model (adding ε shocks has little impact on these variables when $\alpha = 0$). The first thing to note is the nonmonotonicity of K versus ι. The economic logic is nice and easy: on the one hand, higher ι taxes selling k in the DM, and that decreases incentives to invest in the CM; on the other hand, higher ι taxes buying k in the DM, and that increases incentives to invest in the CM; so the net effect depends on ι as well as θ, with the latter especially important because it

[16] It is no surpise that $\bar{\iota}$ increases with θ, but in any case getting $\bar{\iota}$ higher is helped by banking, as discussed above. We could further increase it in various ways, e.g., adding a demand for money by households buying consumption goods, or adding a motive other than reallocation for firms to hold cash.

determines whether buyers or sellers get the lion's share of the bilateral surplus. The impact of ι on K leads to a similar impact on Y and notice two things: the ι where effects turn from negative to positive is empirically relevant; and the effects are not small.[17]

The middle row in Figure 6 focuses on reallocation variables. As in the data, in the long run inflation decreases the R share and increases the P share. Also, the probability Φ of full sales in meetings goes down with ι, which we knew already since $\partial\Phi/\partial\iota < 0$ in Table 5; as accords well with intuition, higher ι makes it more likely that buyers cash out before sellers stock out. Higher ι raises the real DM price $(p + d)/q$, but not much, where the graph show the average price because the model generates price dispersion across meetings, depending on the ε of both buyer and seller. This is consistent with data showing the law of one price is not upheld in used-capital markets (recall fn. 1); as interesting as that may be, detailed analysis of the equilibrium price distribution must be left to future work.

The last row shows average productivity, which falls with ι because that hinder reallocation and keeps more capital in the hands of less productive firms. Also shown is welfare as a function of ι, say $\mathcal{W}(\iota)$, measured as the percent of C agents would give up to reduce ι to 0. As shown, $\mathcal{W}'(\iota) < 0$ and $\iota = 0$ is optimal, but that is not always true (e.g., $\theta = 1$ implies $\iota > 0$ is optimal). A standard summary statistic in the literaure is the welfare cost of 10% inflation, which at $\theta = 1/2$ we compute to be a rather big 2.92.[18] The next chart is money holdings over Y, a standard empirical notion of money demand, where dividing Y is meant to render the time series stationary. It resembles well the money demand curve of firms in the data, and note that we calibrated to the mean but not the elasticity. The final chart shows debt over Y, where debt is d, average non-cash DM payments. It goes up with ι as agents try to switch out of cash and into credit.

[17]While Y and K are nonmonotone, H is increasing in ι for the parameters shown here. As mentioned in fn. 6, in indivisible-labor models people typically call $1 - H$ unemployment, rather than leisure. On that interpretation, we get a stable, long-run Phillips curve, exploitable by policy: unemployment falls with inflation, although not a lot. This contrasts with a related paper, Berentsen et al. (2011), where unemployment increases with ι. the models have many differences, but the key for this is clear: that model has no capital.

[18]This number is virtually identical in the high and low δ calibrations, although as is well understood it is senstive to θ (e.g., Craig and Rocheteau 2008). Thus, for $\theta = 2/3$ it goes down to 1.40% while for $\theta = 1/3$ it goes up to 3.61. As a comparison, in more standard analyses with households using money to buy goods, rather than firms using it to buy capital, roughly speaking reduced-form model, like CIA or MUF models, which seem to only use Walrasian pricing, typically find around 0.5% (e.g., Lucas 2000), while search-and-bargaining models, with θ calibrated, get closer 5.0% (e.g., Lagos and Wright 2005)

Based on all this we think the calibrated model generates several insights into long run issues. One is that investment and output are decreasing in ι when it is low, but increasing when it is moderately higher, while employment is increasing for all ι at calibrated parameters; hence inflation can stimulate investment, output and employment, but that does not mean it is good welfare. Another lesson is that we can get the R and P shares to depend on ι in ways consistent with the long run evidence, which was a prime goal. Given reallocation depends on ι, so does productivity. Another lesson concerns the impact of bargaining on investment, reallocation, money demand, the cost of inflation, etc. which one would miss if never moving beyond Walrasian pricing. Other insights, about search frictions, fiscal policy and productivity persistence, are discussed in Section 6.

5.3 Short-Run Results

The next step is to ask how the model accounts for the business cycle facts. Suppose there are shocks to aggregate productivity A, and potentially also to credit conditions as captured by χ_q, motivated by the earlier discussion and empirical findings. Although of course we can shut down either shock, in general the specification is

$$\begin{aligned} \ln A_t &= \rho_A \ln A_{t-1} + \varsigma_{A,t} \\ \ln \chi_{q,t} - \ln \chi_q &= \rho_\chi \left(\ln \chi_{q,t-1} - \ln \chi_q \right) + \varsigma_{\chi,t}, \end{aligned}$$

where $\varsigma_{A,t} \sim N(0, \sigma_A^2)$ and $\varsigma_{\chi,t} \sim N(0, \sigma_\chi^2)$ are i.i.d. We set $\rho_A = \rho_\chi = 0.825$, as is standard in yearly models, roughly corresponding to 0.95 in quarterly models.

In the spirit of one classic RBC experiment, we set $\sigma_A = 1.68\%$ and $\sigma_\chi = 5.52\%$ to match the volatility of output and the R ratio, then ask how well the model captures the volatility and correlation with output for other variables. Tables 7 and 8 the results for the calibrated with an R share of 30% and 35%, respectively. In each case the first column of numbers contains standard deviations from the data. The second has standard deviations from the model with both shocks, and in parentheses from the model with only A shocks. The third has correlations with output from the data. The last column has correlations with output from the model with both shocks, and in parentheses from the model with only A shocks. Statistics from the model and data are computed identically, as is common, by taking logs and filtering out lower frequencies.

The first observation is that Tables 7 and 8 are very similar, so the results discussed below do not depend much whether the R share is 30% and 35% on average with one exception mentioned below. Hence we focus on Table 7. The next observation is that for the standard macro variables Y, C, I and H the model does a very reasonable job by the standards of the literature accounting for the volatility and correlation with output in the data. Moreover, this is true with only A shocks, and with both A and χ_q shocks – although the numbers in and out of parentheses are not the same, it seems hard to argue that either constitute a better match to data. This is not a big surprise or success; it is more of a minimal requirement, that our new features not impair performance of the textbook model for standard macro data.

Table 7: Business Cycle Statistics

	std. dev. data	std. dev. model	corr. data	corr. model
Output	1.90	1.90 (1.90)	1.00	1.00
Consumption	0.63	0.50 (0.60)	0.91	0.87 (0.97)
Investment	3.74	3.44 (3.04)	0.86	0.98(0.99)
Employment	0.74	0.61 (0.45)	0.90	0.91 (0.94)
TFP	0.62	0.74 (0.80)	0.79	0.98 (0.99)
R share	5.88	5.88 (2.32)	0.59	0.31 (-0.97)
P share	9.51	10.14 (0.48)	-0.50	-0.56 (0.13)
Inflation	1.93	3.83 (0.07)	0.35	0.44 (-0.98)

Note: std. dev. is relative to output, except for output itself; corr. is correlation with output..
In brackets are numbers for the case of only productivity shocks.

Next, consider reallocation dynamics. On that dimension, the model with only A shocks is way off. For the R share, the standard deviation in parentheses is too small, and correlation with output is -0.97 instead of $+0.59$, which is a rather egregious miss. For the P share, if far too small, 0.48 compared to 9.51 in the data, and the correlation with output again takes the wrong sign. Heuristically, positive A shocks make low productivity firms more demanding when selling k, which reduces reallocation. The conclusion is clear: with only productivity shocks the model cannot capture reallocation dynamics at all.

Now consider reallocation dynamics with χ_q shocks, as well as A shocks, given by the numbers not in parentheses. These are better, and indeed match the data remarkably

Table 8: Business Cycle Statistics: Alternative Calibration

	std. dev. data	std. dev. model	cor.r data	corr. model
Output	1.90	1.90 (1.90)	1.00	1.00
Consumption	0.63	0.41 (0.53)	0.91	0.83 (0.97)
Investment	3.74	3.90 (3.36)	0.86	0.98(0.99)
Employment	0.74	0.70(0.50)	0.90	0.94 (0.96)
TFP	0.62	0.67 (0.76)	0.79	0.98 (0.99)
R share	5.88	5.88 (2.37)	0.59	0.46 (-0.98)
P share	9.51	12.95 (0.42)	-0.50	-0.67 (0.10)
Inflation	1.93	4.33 (0.06)	0.35	0.63 (-0.99)

Note: std. dev. is relative to output, except for output itself; corr. is correlation with output.. In brackets are numbers for the case of only productivity shocks.

well. Of course it is no surprise it matches the volatility of the R share: σ_χ is calibrated to that. We did not calibrate to match the correlation between the R share and output, compared to the model with only A shocks, the magnitude is in the ballpark and at least the sign is not wrong.[19] Moreover, for the P share the standard deviation and correlation with output in the model are very close to the data, while with only A shocks while the sign and magnitude were way off. This is all consistent with the economic intuition laid out above, but it is important to see how it work quantitatively.

While the results in the previous paragraph are what we want to emphasize, there are a few subsidiary results. With only A shocks, inflation is strongly countercyclical, the opposite of the data, and its volatility is too small, while with both shocks the model gets the sign right for the correlation with output, and the volatility is more reasonable (at least it is positive, if a bit too big). Again this is consistent with intuition, although here we are less interested in inflation dynamics, given the setup (e.g., we have a fairly simplistic specification for monetary policy). In any case, to summarize, we conclude that the model is successful at matching the reallocation observations, and some other facts, with no compromise in performance on standard macro facts.

[19]This is the one dimension where one might say the two calibrations are a little different: the correlation with between the R share and Y is 0.59 in the data, and the result from the model in Table 7 is a reasonable 0.31, while in Table 8 it is even better, coming in at 0.46. We do not consider than a major difference.

6 Extensions and Other Applications

Here we take up the impact of search efficiency, fiscal policy, and persistence in firm-specific shocks.

6.1 Effects of Search in Different Specifications

Since search figures prominently in the paper, and in many of the others mentioned in the Introduction, one might like to know how equilibrium depends on search efficiency as measured by α. Additionally, given that a version without ε shocks is the textbook model, with no need for the DM or money, we can starting from that and move to the benchmark model gradually by including ε shocks, and then other components, one at a time. Here we concentrate these versions: (i) search frictions in the DM, but perfect credit, so money is not needed; (ii) add to this constrained credit, but still no money; and (iii) our main model with money.[20]

The first finding is that in response to aggregate shocks all the versions generate statistics for standard macro variables that are about the same, i.e., close to the textbook model. We do not show analogs to Table 7 (they can be made available on request) for each specification, because the results are so similar, and because they are not too surprising, but it is good to verify that adding firm-specific shocks in different ways does not harm performance in accounting for the main macro facts. However, in terms of reallocation, the specifications are quite different. An obvious difference is that with perfect credit there are no partial sales, making it a nonstarter for explaining the two types of reallocation, but we can still looks at its properties on other dimensions.

This is displayed in Figure 7 as α varies, for different values of θ. As shown, with perfect credit, for small θ higher α increases CM investment and output, but as θ increases the effects fall and eventually turn negative. This makes sense: as in the benchmark model, higher α means better DM trading opportunities; if θ is low, that mostly benefits sellers

[20] We also tried versions where sellers in the DM trade with probability α, but when they trade, the highest ε firm gets all the capital of the others, which can be interpreted as having search but not matching problems. We also tried that with $\alpha = 1$ to eliminate search, which makes the DM completely frictionless if we have perfect credit and Walrasian pricing; in the interest of space those results are omitted (but they can be made available on request).

so agents invest more in the CM; but if θ is high, it mostly benefits buyers, so agents invest less in the CM. In any case, higher α always increases welfare and productivity, with welfare highest at $\theta = 0$. Since there is perfect credit, θ does not affect DM trade – that always entails full sales – but it affects how agents split the gains form trade and that affects CM investment.

Moving to imperfect credit, but still no money, we need $\chi_k > 0$ for DM trade, and set it to 0.05 to generate some full and some partial sales. As shown in Figure 8, the results are different than those the case of perfect credit. Now higher θ raises output, consumption and welfare, while in Figure 7 higher θ lowers output, consumption and welfare. Also, investment falls with α for the values of θ considered, but with high θ productivity rises sufficiently fast with α to offset the fall in investment, leading to more output. One thing these two variants have in common is all variables are monotone in α, and the value of α does not change the ordering of variables across different θ.

Things are even more different in the monetary money. As shown in Figure 9, output and consumption now can be nonmonotone in α. Moreover, the R ratio and welfare are maximized at different θ depending on α. If α is large, $\theta = 0.5$ yields higher welfare and a higher R ratio than when θ is small, different from the nonmonetary models. Intuitively, endogenous liquidity introduces a trade-off: higher θ induces agents to hold more Z, which is good; but higher θ also exacerbates the holdup problem on sellers as they internalize less of the surplus from CM investment, which is bad. In general, there is a lot more going on in Figure 9 than in the nonmonetary models. What we take away from this is that search, matching and bargaining are interesting in models of secondary capital markets, *especially* in monetary economies.

6.2 Fiscal Policy

Now consider taxation. While there are various experiments one could run, Figure 10 plots the usual slate of variables against τ_k, over a range of values, from the calibrated $\tau_k = 0.25$ down to large negative values. Capital taxation has big effects on the standard macro variables, but that is true in most models (e.g., McGrattan et al. 1997). Interestingly, τ_k hardly matters for reallocation, in the sense that the P and R shares are flat;

that means the absolute amount of reallocation changes a lot, but the R share is approximately constant as investment moves with τ_k. We do see a noticeable impact on money demand, presumably because reallocating capital is less lucrative, and hence liquidity is less important, when the income is more heavily taxed.

In these experiments, when τ_k changes any impact on public revenue is made up by adjusting the lump sum tax T. That is perhaps not too realistic, although it is still an interesting exercise. To pursue it further, recall from the theoretical analysis that (26) gives the τ_k^* that supports the first best, for any θ, when other fiscal and monetary distortions are eliminated with $\tau_h = 0$ and $\iota \to 0$. Putting numbers on it, for $\theta = (0, 0.25, .0.5, .0.75, 1)$ the optimal subsidy (i.e., $-\tau_k^*$) is in percentage terms is $(0, 1.2, 2.42, 3.7, 4.95)$. Obviously higher θ implies a bigger subsidy because there is a bigger holdup problem when selling capital in the DM, but even at $\theta = 1$ the subsidy is less than 5%.

In addition to having access to the lump sum T to sterilize changes in τ_k, the above scenario is also extreme because it concerns the fully optimal policy, with $\tau_h = 0$ and $\iota \to 0$. Consider instead the optimal subsidy at the calibrated values of τ_h and ι. Ignoring $\theta = 0$, because then there is no monetary equilibrium at $\iota > 0$, we report $\theta = (0.33, 0.5, 0.66)$ implies $\tau_k^* = (54.0, 56.0, 58.0)$. These are an order of magnitude bigger than with $\tau_h = 0$ and $\iota \to 0$, because of the additional distortions including the double holdup problem, but it also seems to matter a lot whether $\tau_h < 0$ or $\tau_h = 0$. Also notte that τ_k^* now not at all sensitive to θ in this experiment.

As we said, capital taxation is known to be severely distortionary for the levels of standard macro variables, although it has not been analyzed in models with secondary as well as primary capital markets. We learn from this exercise that θ matters a lot at the fully-optimal policy, with $\tau_h = 0$ and $\iota \to 0$, yet it barely matters at all with $\tau_h > 0$ and $\iota > 0$ set to realistic levels. Pursuing fiscal implications further would be interesting, perhaps solving a Ramsey optimal tax problem. Aruoba and Chugh (2010) show the solution qualitatively and quantitatively change when one tries to model liquidity with better microfoundations, and it would be interesting to see how that plays out in economies with capital reallocation.

6.3 Persistent Shocks

Above we used i.i.d. firm-specific shocks. Now suppose ε can be decomposed into a persistent component a and a transient component ϵ,

$$\log \varepsilon = \log a + \log \epsilon,$$

Here $a \in \{1-x, 1+x\}$, with $x \in [0,1)$, so a is a two-state Markov process, and we assume $\log \epsilon$ is independent of $\log a$ and i.i.d. normal. The generalization to an N-state process is straightforward in principle but computationally more intense (and it is already intense because it involves a lot of numerical integration). Firms' (k, z) choice in the CM depend on the persistent component a. We are interested in how differences in x, and hence gap between high and low a, affects the economy. To this end, we keep all previous parameters the same and use switching probability $1-p$ to get a stationary distribution where half of the firms have $a = 1-x$ and rest have $a = 1+x$.

Setting $p = 0.75$, we can vary x without changing average productivity, but making high a firms more productive than low a firms. Results are shown in Figure 11, where the horizontal axis is $2x$, measuring the productivity gap between high a and low a firms. Consider first the impact on aggregate variables. As x increases, output, investment and employment all increase, capturing the what might be called the option value of reallocation: because firms with low productivity can sell their k to high productivity firms, albeit in a frictional market, more dispersion is good. The effects are sizable: Y increases about 1% if we change x from 0 to 0.05, which means average productivity of the high a firms is 10% higher than low a firms.

Higher x also increases capital holding for high a firms and reduces it for low a firms. Intuitively, higher x implies the high a firms are more productive and hence want more k, while low a firms want less. As for money holdings, higher x reduces z for high a firms. The reason is that low a firms hold less k, so buying k with cash in the DM is less profitable. Money holdings of low a firms is non-monotone in x: if x is small, their z increases with x, because while low a firms are less productive, they will want to acquire a lot of k if their transient shock is big, so they hold more z; but if x is large there is not much gain from reallocation because their productivity is unlikely to be high and hence their demand for z

falls with x. In aggregate, total money demand decreases with x.

The above results concern levels for aggregate macro variables. One can also look at how x affects second moments. It turns out that for the standard macro variables these do not depend a lot on x. This is reminiscent of Rios-Rull (1996), who asks how results in models like Kydland and Prescott (1982) change when we incorporate realistic life cycles. He shows the responses of aggregate variables to aggregate shocks do not change much, which is convenient since representative-agent models are much easier to solve; but note that life cycle models generate more than aggregate statistics – e.g., cross sections due to age heterogeneity.

Analogously, our model with persistent shocks generates aggregate statistics for standard variable that are to the i.i.d. case, which is convenient as the latter is much easier to solve; but it also generates more statistics – e.g., a cross section of k and z. On that our setup with heterogeneity generates more statistics. In principle one could try to match the distribution of k and z across firms and how that varies over the business cycle. We looked at that, but to get something empirically reasonable it seems important to go beyond a two-state Markov process for the persistent component a; right now the model generates a two point distribution for k and z at the end of the CM, and while these spread out after the DM due to reallocation, the result naturally tends to be bimodal. Going beyond a two-state process seems promising but it as mentioned it is computationally intense and hence we leave it to future research.

7 Conclusion

This paper has developed a model consistent with the empirical relationships we found between different types of capital reallocation and the cost of liquidity measured by inflation. The theory predicts that a higher cost of liquidity reduces full sales and increases partial sales. That captures the long-run patterns in the data. Then we added shocks to credit conditions. Easier credit reduces the demand for money, increasing inflation in the short run, while increasing full sales and decreasing partial sales. That captures the business-cycle patterns in the data.

The framework also provided insights into how fiscal policy and frictions related to search, matching and bargaining affect reallocation, and the impact on the macroeconomy. Additionally, it allowed us to study how persistence in productivity shocks affects firms' capital and liquidity positions. The model generated fairly strong analytic results on existence, uniqueness, and comparative statics for steady states, and proved amenable to calibration using mostly standard observations. This suggests there may be more applications for this and similar models in future research.

Appendix A: More on Data

Financial data are from the Flow of Funds Accounts (Z1 Report of the Federal Reserve Board). We use the Coded Table released on December 8, 2016; new editions may use different coding, so one should take that into account. Total business (corporate and non-corporate) debt in nominal terms is the sum of Debt Securities (Table F.102, item 30) and Loans (Table F.102, item 34), with the GDP implicit price deflator (Table 1.1.9 in NIPA) used to put this in 2009 dollars. Aggregate consumption and investment in 2009 dollars are from Table 1.1.3 in NIPA, excluding residential investment, consumer durables, government expenditures and net exports. The AAA corporate (nominal) bond yield and (CPI) inflation are from FRED at the St. Louis Fed.

For capital reallocation, COMPUSTAT (North America) contains useful information on ownership changes of productive assets starting in 1971. We measure capital reallocation by sales of property, plant and equipment (SPPE, data item 107 with combined data code entries excluded), plus full buyouts (AQC, data item 129 with combined data code entries excluded) from 1971 to 2018. We also use capital spending (CAPX, data item 128). Since capital spending in COMPUSTAT excludes full sales, the level of capital expenditures of each firm is calculated as the sum of AQC and CAPX. Finally, we mention for the empirical work on micro data in Section 2, industries are excluded with standard industry classification (SIC) codes below 1000 (agriculture, forestry and fishing), above 9000 (public and non-classified), and between 6000 and 6500 (financial).

Appendix B: Proofs

Proof of Proposition 1. Clearly, either the constraints on p and d both bind or they are both slack. Suppose they bind, and consider solving (12) ignoring the constraint on q. The Kalai condition $(1-\theta)\, S^b(\mathrm{s}, \tilde{\mathrm{s}}) = \theta S^s(\tilde{\mathrm{s}}, \mathrm{s})$ yields $q = Q$. If $Q < \tilde{k}$ then the true (i.e., constrained) solution is $q = Q$ and the constraints on p and d at equality, which is case (i). If $Q > \tilde{k}$ then the true solution is $q = \tilde{k}$ and the Kalai condition gives the total payment, which is case (ii). Finally, the threshold in (13) comes from rearranging $Q < \tilde{k}$. ■

Proof of Proposition 2. Consider the IS curve. If $L \leq \underline{L} \equiv (1-\theta)\, \varepsilon_L$, the integral in (23) is 0, and

$$B = \overline{B} \equiv \frac{r+\delta}{(1-\tau_k)\mathbb{E}\varepsilon + (1-\delta)\,\chi_k \iota}.$$

The IS curve is decreasing and $B \to \widetilde{B}$ as $L \to \infty$, where

$$\widetilde{B} \equiv \frac{r+\delta}{(1-\tau_k)\left[\mathbb{E}\varepsilon + \alpha\,(1-\theta) \iint\limits_{\varepsilon<\tilde{\varepsilon}} (\tilde{\varepsilon}-\varepsilon)\,dF(\tilde{\varepsilon})dF\,(\varepsilon)\right]}.$$

Intuitively, if L is larger the liquidity constraint is looser and opportunities for resale are better, so firms invest in more k even if the benefit from production B is low.

Now consider LM. If $L \leq \underline{L}$, buyers are constrained in all transactions and $B = \underline{B}$ where $\underline{B}$ solves

$$\iota = \iint\limits_{\varepsilon>\tilde{\varepsilon}} \frac{\alpha B\theta(1-\tau_k)\,(\varepsilon-\tilde{\varepsilon})}{\Delta\,(\varepsilon,\tilde{\varepsilon})} dF(\tilde{\varepsilon})dF\,(\varepsilon)\,.$$

Intuitively, $\underline{B}$ increases with ι and $B = 0$ at $\iota = 0$. As L increases, buyers become less constrained. To make them willing to hold money it must be that the benefit B from reallocation is higher. Notice $B \to \infty$ as $L \to \tilde{L}$, where $\tilde{L}$ solves

$$\iota = \iint\limits_{\mathcal{S}_1(\tilde{L})} \frac{\alpha\theta\,(\varepsilon-\tilde{\varepsilon})}{(1-\theta)\,\varepsilon+\theta\tilde{\varepsilon}} dF(\tilde{\varepsilon})dF\,(\varepsilon)\,.$$

This implies that when a monetary steady state exists, it uniquely pins down B and L, and they uniquely determine w and Z/K. It remains to show K is unique. By the definition of $J\,(L,w)$ and (23), $J\,(L,w)\,B\,(w) \geq (r+\delta)\,/\,(1-\tau_k) > r$. Then there is a unique $K > 0$ solving (25), finishing the uniqueness result. Existence is standard in this kind of model (e.g., see Gu and Wright 2016), so the details are omitted. ■

Proof of Proposition 3. We solve the planner problem given the DM frictions. First note that in the CM labor should be allocated to firms according to

$$h^*(k,\varepsilon) = \left[\frac{\eta u'\,(c)}{\xi}\right]^{\frac{1}{1-\eta}} A\varepsilon k. \tag{27}$$

Aggregating across firms gives total hours, and $h \leq 1$ is assumed slack. Also, when two firms meet in the DM the higher ε firm should get all the capital. Given these observations, consider a planner choosing a path for k to maximize utility of the representative agent, subject to an initial k_0 and resource feasibility after government takes G_t units of x. Assuming ε is i.i.d., for simplicity, $\hat{k}$ is the same for all agents in the CM.

Then the problem can be written

$$
\begin{aligned}
W^*(k_0) &= \max_{k_{t+1}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \xi h_t\right] \qquad (28) \\
\text{st } c_t &= y_t + (1-\delta) k_t - G_t - k_{t+1} \\
y_t &= (1-\alpha) \int_0^{\infty} (A\hat{\varepsilon}k_t)^{1-\eta} h^*(k_t,\hat{\varepsilon})^{\eta} dF(\hat{\varepsilon}) \\
&\quad + \alpha \int_{\hat{\varepsilon}>\tilde{\varepsilon}} (A\hat{\varepsilon}2k_t)^{1-\eta} h^*(2k_t,\hat{\varepsilon})^{\eta} dF(\hat{\varepsilon}) \, dF(\tilde{\varepsilon})
\end{aligned}
$$

where output y_t includes production by the $1-\alpha$ measure of firms that did not have a DM meeting, the α measure that had a meeting and increased k, plus the α measure that had a meeting and decreased k. Routine methods yield the Euler equation

$$
\begin{aligned}
r_t + \delta = (1-\alpha) &\int_0^{\infty} (1-\eta)(A\hat{\varepsilon})^{1-\eta} \left[\frac{h^*(k_{t+1},\hat{\varepsilon})}{k_{t+1}}\right]^{\eta} dF(\hat{\varepsilon}) \qquad (29) \\
+ 2\alpha &\int_{\hat{\varepsilon}>\tilde{\varepsilon}} (1-\eta)(A\hat{\varepsilon})^{1-\eta} \left[\frac{h^*(2k_{t+1},\hat{\varepsilon})}{2k_{t+1}}\right]^{\eta} dF(\tilde{\varepsilon})\, dF(\hat{\varepsilon})
\end{aligned}
$$

where r_t satisfies $1 + r_t = u'(c_t)/\beta u'(c_{t+1})$.

Next, use (27) to write

$$
r_t + \delta = (1-\eta) A \left[\frac{\eta u'(c_{t+1})}{\xi}\right]^{\frac{\eta}{1-\eta}} \left[\int_0^{\infty} \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha \int_{\hat{\varepsilon}<\tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon})\, dF(\tilde{\varepsilon})\, dF(\hat{\varepsilon})\right]. \qquad (30)
$$

Recall that in equilibrium with $\iota \to 0$, $\tau_k = \tau_h = 0$, transactions are efficient in the DM and the Euler equation for k_t is

$$
r_t + \delta = (1-\tau_k) B(w_{t+1}) \left[\int_0^{\infty} \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha(1-\theta) \int_{\hat{\varepsilon}<\tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon})\, dF(\tilde{\varepsilon})\, dF(\hat{\varepsilon})\right],
$$

where $1 + r_t = u'(c_t)/\beta u'(c_{t+1})$, $B(w) = (\eta/w)^{\frac{\eta}{1-\eta}}(1-\eta) A$ and $u'(c) = \xi/(1-\tau_h) w$. Therefore, in equilibrium

$$
\begin{aligned}
r_t + \delta &= (1-\eta)(1-\tau_k) A \left[\frac{\eta(1-\tau_h) u'(c_{t+1})}{\xi}\right]^{\frac{\eta}{1-\eta}} \\
&\quad \left[\int_0^{\infty} \hat{\varepsilon} dF(\hat{\varepsilon}) + \alpha(1-\theta) \int_{\hat{\varepsilon}<\tilde{\varepsilon}} (\tilde{\varepsilon} - \hat{\varepsilon})\, dF(\tilde{\varepsilon})\, dF(\hat{\varepsilon})\right].
\end{aligned}
$$

Comparing this with (30), one can see that $\theta > 0$ implies agents do not fully internalizes the benefits of investment, so there is under accumulation of capital under under at $\tau_k = \tau_h = 0$. But if $\tau_h = 0$ and τ_k is given by (26, the first best is achieved. ■

Proof of Proposition 4. (23) defines a unique k for any $w \in (\underline{w}, \bar{w})$, where

$$B(\underline{w}) = \frac{r+\delta}{\mathbb{E}\varepsilon(1-\tau_k)},\ B(\bar{w}) = \frac{r+\delta}{[\mathbb{E}\varepsilon + \alpha(1-\theta) I_s(\infty)](1-\tau_k)}.$$

Suppose that ε is bounded from 0 and infinity. If $w = \underline{w}$, any sufficiently large K solves (23). If $w = \bar{w}$, any sufficiently small K solves (23). Also, (25) implies w is increasing in k. Moreover $w = 0$ if $k = 0$ and k is finite if $w = \bar{w}$. By continuity there is a steady state. If α is not too big, the curve defined by (23) is decreasing in w, implying uniqueness. ■

Comparative statics wrt ι: If $\theta = 1$ then

$$\frac{r+\delta}{B(w)(1-\tau_k)} = \mathbb{E}\varepsilon + \chi_k \iota; \tag{31}$$

$$\iota = \iint_{\mathcal{S}_1(L)} \frac{\alpha(\varepsilon - \tilde{\varepsilon})}{(1-\theta)\varepsilon + \theta\tilde{\varepsilon} + (1-\delta)\frac{(1-\chi_q)}{(1-\tau_k)B}} dF(\tilde{\varepsilon})dF(\varepsilon); \tag{32}$$

$$u'^{-1}\left[\frac{\xi}{(1-\tau_h)w}\right] + G = \left[\frac{B(w)J(w,L)}{1-\eta} - \delta\right]K. \tag{33}$$

Hence $B(w)$ is uniquely determined by (31), which determines w and c. Then L is determined by (32). If ι increases, both B and L decrease. Therefore, w increases and Φ increases if θ is not too small. Moreover, if $\chi_k = 0$ then $B(w)$ is constant and $J(w,L)$ decreases. To see this, first rewrite

$$J(L,w) \equiv \int \varepsilon dF(\varepsilon) + \alpha \iint_{\varepsilon > \tilde{\varepsilon}} (\varepsilon - \tilde{\varepsilon}) \min\left\{1, \frac{Z+\chi_0}{\Delta(\varepsilon,\tilde{\varepsilon})K}\right\} dF(\varepsilon)dF(\tilde{\varepsilon}). \tag{34}$$

Recall $\Delta(\varepsilon,\tilde{\varepsilon})$ involves only B and hence does not change. Therefore, $J(L,w)$ decreases if L decreases because

$$L \equiv \frac{(Z+\chi_0)/K - (1-\delta)(1-\chi_q)}{(1-\tau_k)B(w)}.$$

As a result,

$$K = \frac{u'^{-1}\left[\frac{\xi}{(1-\tau_h)w}\right] + G}{\frac{B(w)J(w,L)}{1-\eta} - \delta}$$

increases because w does not change with ι and J decreases. Constant w implies constant c. Then higher K implies a higher Y. Moreover, $(Z+\chi_0)/K$ decreases with ι because L decreases with ι and B is unchanged. By continuity, K and Y increase, $(Z+\chi_0)/K$ decreases with ι if θ is close to 1 and χ_k not too big.

Comparative statics wrt χ_0: As χ_0 does not affect (31)-(33), w, K, Y and L stay the same. Therefore $(Z+\chi_0)/K$ is constant. If χ_0 increases, Z decreases.

Comparative statics wrt χ_k: Higher χ_k shifts the IS curve down and does not affect the LM curve. Hence B and L decrease, so w increases. If $\chi_q = 1$ then L stays constant and B decreases. Thus w increases and K increases, and Y increases because both c and K increase. By continuity, the same is true if χ_q is not too small.

Comparative statics wrt χ_q: If χ_q increases LM shifts down and IS stays the same. Hence L increases and B decreases, w goes up and Φ increases. If θ is close to 1, the change in B is close to 0. As a result, w and c are almost unchanged. At the same time, $B(w)\,J(w,L)/(1-\eta)$ increases because L increases, so K and Y decrease. Because L increases, so does Φ.

Comparative statics wrt τ_k: This shifts up both LM and IS, so $B(w)$ increases, L increases if χ_q close to 1, and w decreases. So Φ increases, and since χ_q is close to 1, $B(w)\,J(w,L)/(1-\eta)$ increases, so K, c and Y decrease.

Comparative statics wrt τ_h: This does not change B or L, so w and Φ stay the same, while c decreases. Then K decreases, which implies Y decreases. Also, Z decreases. By continuity, this holds for large $\theta < 1$.

Appendix C: Multiple Liquid Assets

As in Lester et al. (2012), in addition to z there is a long-lived real asset a in fixed supply 1, with CM price ψ and dividend ρ. Both z and a are used for DM payments, but we allow general χ_z and χ_a, and set $\chi_q = 1$ and $\chi_k = 0$ to ease notation. Then

$$\begin{aligned} W(\Omega,\varepsilon) &= \max_{c,h,\hat{k},\hat{z}} \left\{ u(c) - \xi h + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} V_+(\hat{a},\hat{k},\hat{z},\hat{\varepsilon}) \right\} \\ \text{st } c &= \Omega + (1-\tau_h) w h - \hat{z}\phi/\phi_+ - \hat{k} - \psi \hat{a}, \end{aligned}$$

where $\Omega = (1-\tau_k)B(w)\varepsilon k + (1-\delta)k + z - d - T + (\psi+\rho)\,a$. FOCs are

$$\begin{aligned} -\frac{\xi}{(1-\tau_h)w} + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} \frac{\partial V_+(\hat{a},\hat{k},\hat{z},\hat{\varepsilon})}{\partial \hat{k}} &\leq 0, \ = 0 \text{ if } \hat{k} > 0 \\ -\frac{\xi}{(1-\tau_h)w}\frac{\phi}{\phi_+} + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} \frac{\partial V_+(\hat{a},\hat{k},\hat{z},\hat{\varepsilon})}{\partial \hat{z}} &\leq 0, \ = 0 \text{ if } \hat{z} > 0 \\ -\frac{\xi}{(1-\tau_h)w}\psi + \beta \mathbb{E}_{\hat{\varepsilon}|\varepsilon} \frac{\partial V_+(\hat{a},\hat{k},\hat{z},\hat{\varepsilon})}{\partial \hat{z}} &\leq 0, \ = 0 \text{ if } \hat{a} > 0. \end{aligned}$$

In the DM there are three types of meetings: with probability α_1 only z is accepted; with probability α_2 only a is accepted, and with probability α_3 both are accepted. Emulating the methods in the text, we arrive at

$$\begin{aligned}
\frac{r+\delta}{B(w)(1-\tau_k)} &= \mathbb{E}\varepsilon + (1-\theta)\left[\alpha_1 I_s(L_1) + \alpha_2 I_s(L_2) + \alpha_3 I_s(L_3)\right]. \\
\iota &= \alpha_1 \chi_z \lambda(L_1) + \alpha_3 \chi_z \lambda(L_1 + L_2) \\
r Z_a &= (1+r)\chi_a \rho + \beta Z_a \chi_a \left[\alpha_2 \lambda(L_2) + \alpha_3 \lambda(L_1 + L_2)\right]
\end{aligned}$$

where $Z_a = (\rho + \psi)\chi_a$,

$$\lambda(L) \equiv \iint\limits_{\mathcal{S}_b(L)} \frac{\alpha\theta(\varepsilon - \tilde{\varepsilon})\, dF(\tilde{\varepsilon}) dF(\varepsilon)}{(1-\theta)\varepsilon + \theta\tilde{\varepsilon}},\ L_1 \equiv \frac{\chi_z Z}{(1-\tau_k) B(w) K},\ L_2 \equiv \frac{\chi_a Z_a}{(1-\tau_k) B(w) K},$$

Here L_1 and L_2 are liquidity per unit of effective capital. This determines the fraction of unconstrained meetings and the fraction of capital traded in constrained meetings. If $\theta = 1$, B is constant and L_1 and L_2 are proportional to the per capital liquidity.

$$\begin{aligned}
\iota &= \chi_z \left[\alpha_1 \lambda(L_1) + \alpha_3 \lambda(L_1 + L_2)\right] \\
r &= C + \chi_a \left[\alpha_2 \lambda(L_2) + \alpha_3 \lambda(L_1 + L_2)\right],
\end{aligned}$$

where $C = (1+r)\chi_a \rho/(1-\tau_k) B(w) K L_2$. If ρ is close to 0 so are $\partial C/\partial L_2$ and $\partial C/\partial L_2$.

Because $\lambda' < 0$, we have

$$\begin{aligned}
\frac{dL_1}{d\iota} &\approx \frac{\partial C}{\partial L_2} + \chi_a \left[\alpha_2 \lambda'(L_2) + \alpha_3 \lambda'(L_1 + L_2)\right] < 0 \\
\frac{dL_2}{d\iota} &\approx \frac{\partial C}{\partial L_1} - \chi_a \alpha_3 \lambda'(L_1 + L_2) > 0.
\end{aligned}$$

Therefore, if ρ is not too big

$$\frac{d(L_1 + L_2)}{d\iota} \approx \chi_a \alpha_2 \lambda'(L_2) + \frac{dC}{dL_2} - \frac{\partial C}{\partial L_1} < 0.$$

As ι increases, liquidity per productive unit of k falls. By continuity, if $\theta < 1$ is not too small and $\rho > 0$ is not too big the same result holds. Exactly as suggested by Wallace (1980), inflation reduces total liquidity even if it directly taxes only z, and not a, due to general equilibrium effects as agents try to move out of z and into a.

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Figure 1: Reallocation and the Cost of Liquidity

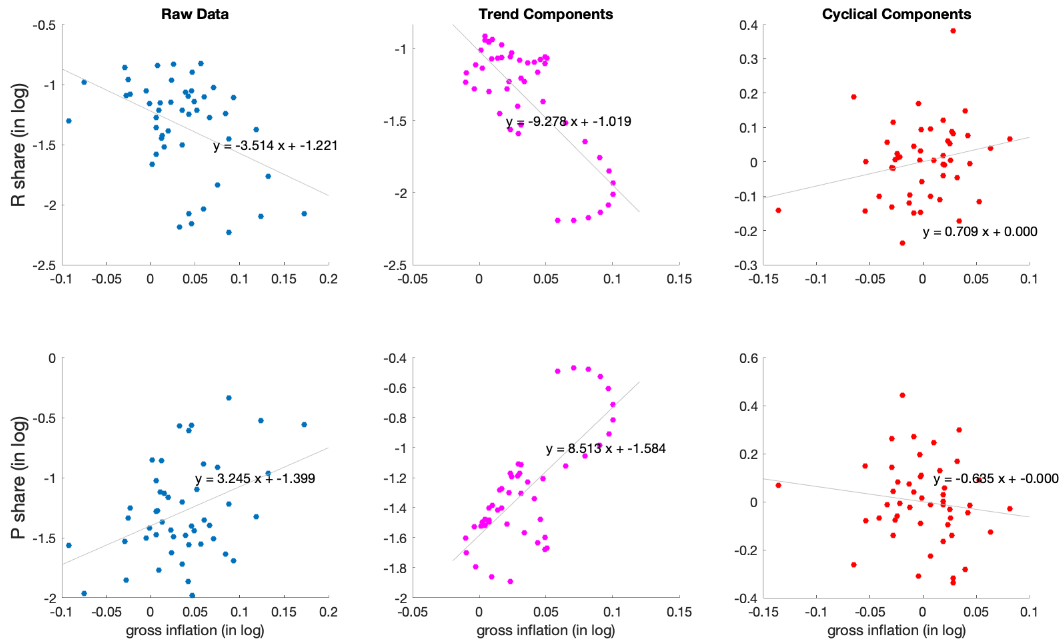
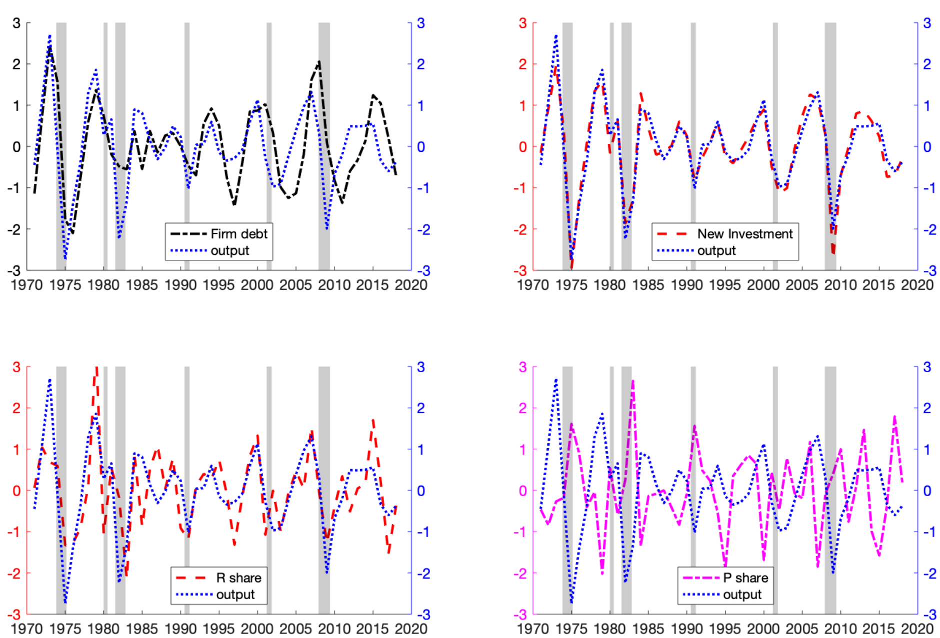


Figure 2: Debt, Investment and Reallocation



Note: shaded areas denote NBER recession dates.

Figure 3: Events in Period t

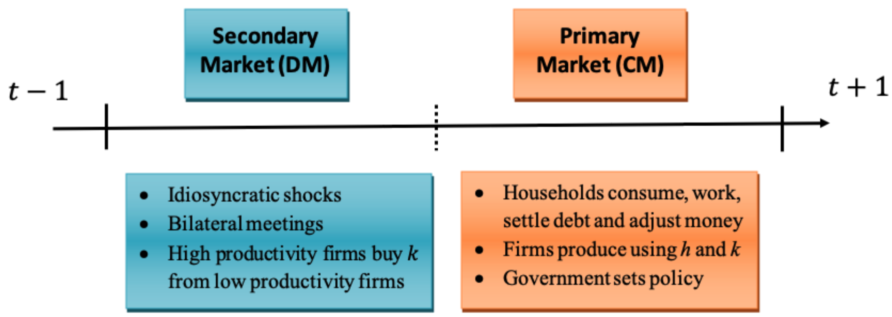


Figure 4: DM Trade: Stock Out or Cash Out

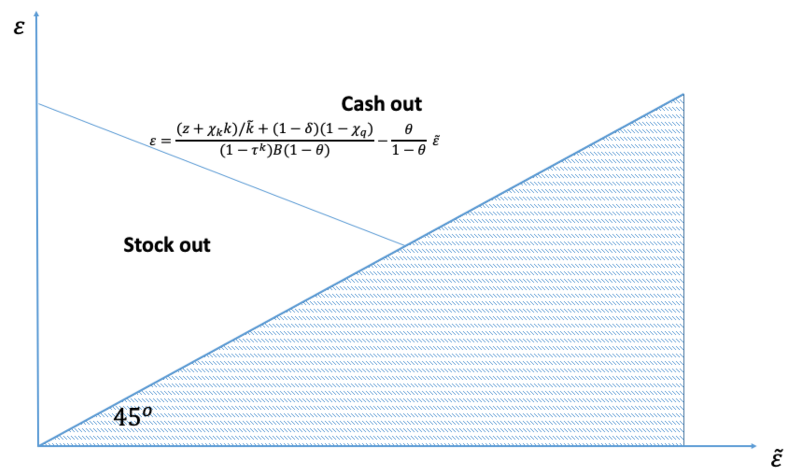


Figure 5: Steady-State Equilibrium

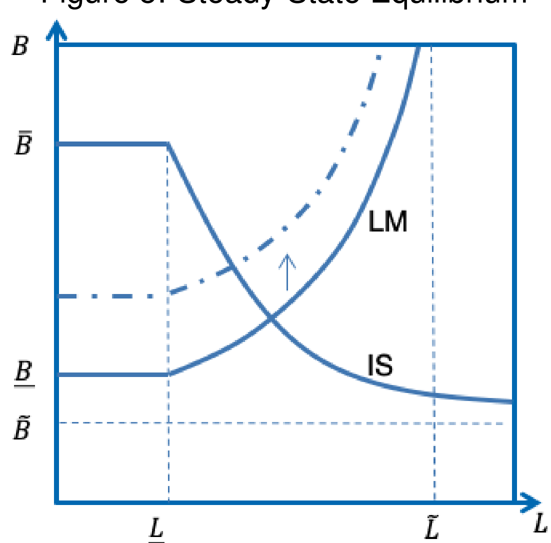
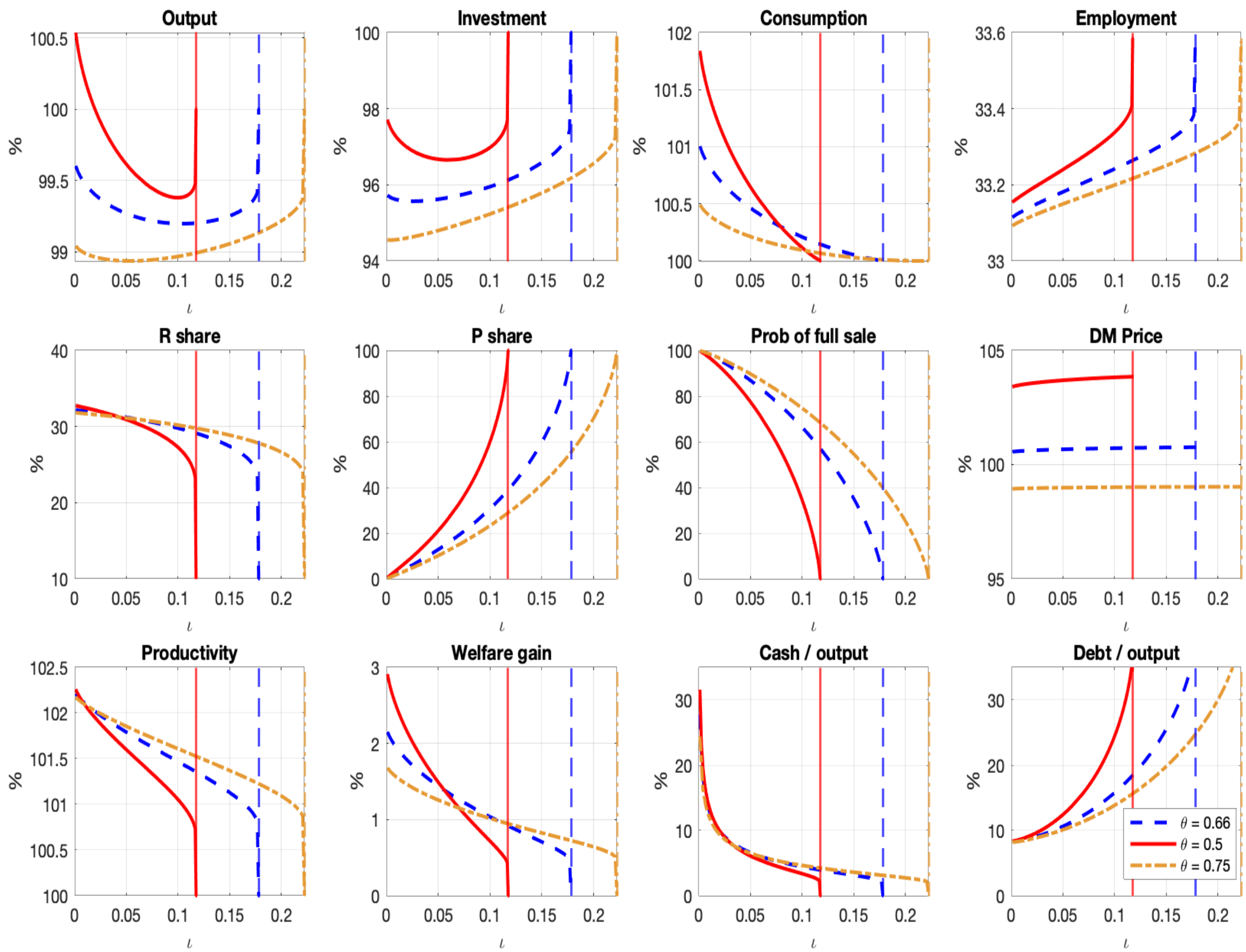
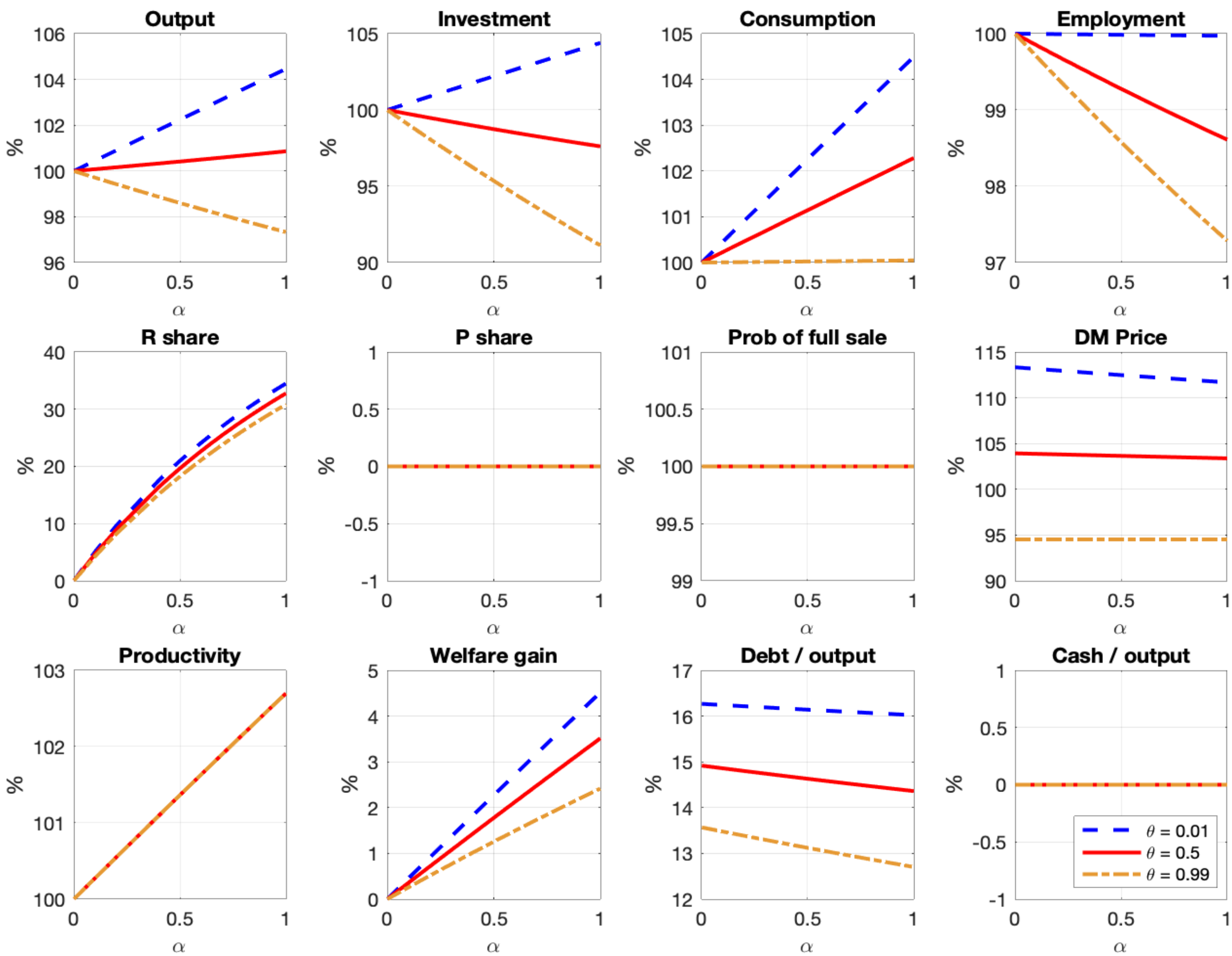


Figure 6: The Long-Run Effects of Inflation



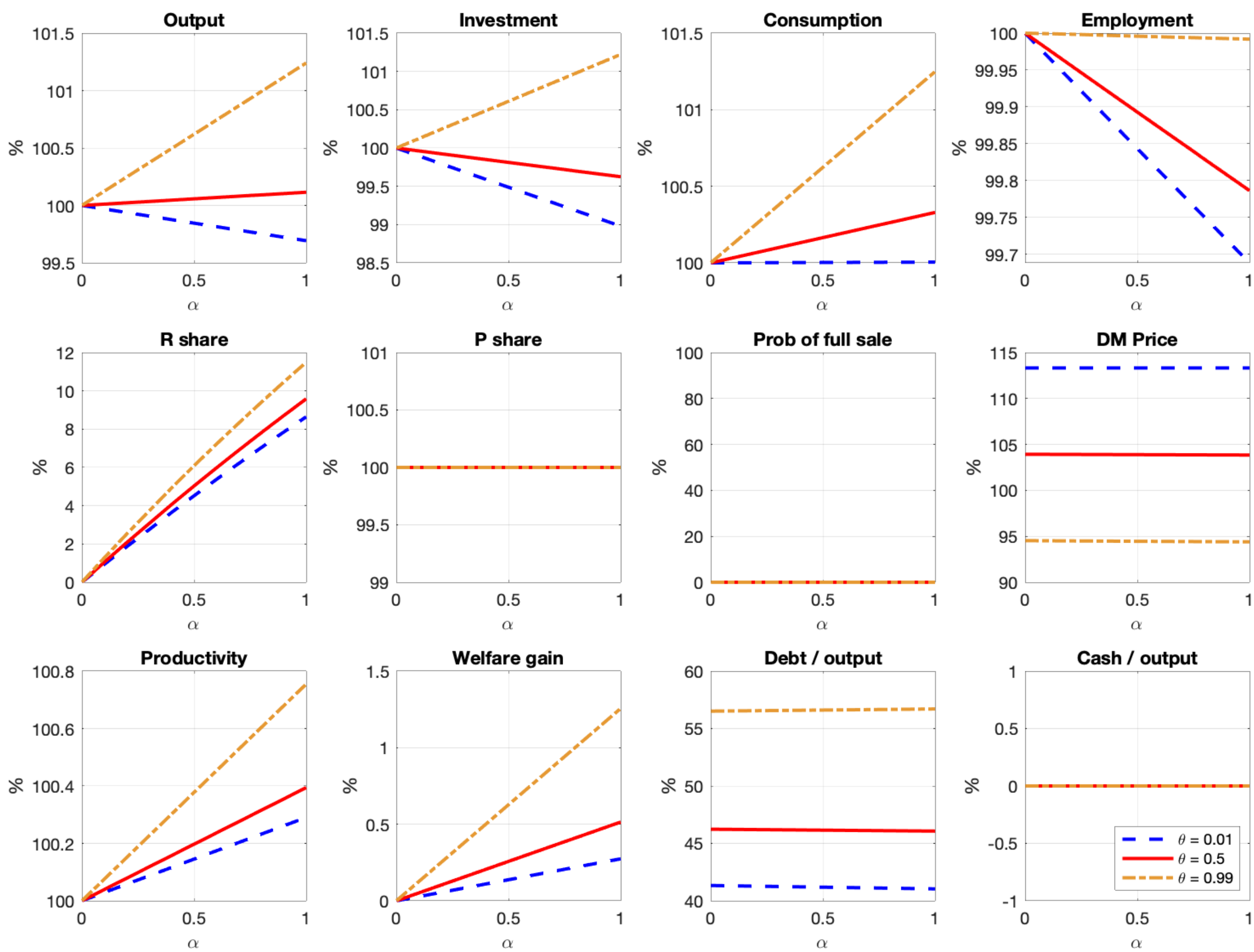
Note: Each vertical line represents the boundary of the monetary region for a given θ. For output, investment, consumption, productivity, and welfare, the corresponding levels in the non-monetary region are used as the normalization.

Figure 7: Long-run Effects of Search Frictions with Perfect Credit



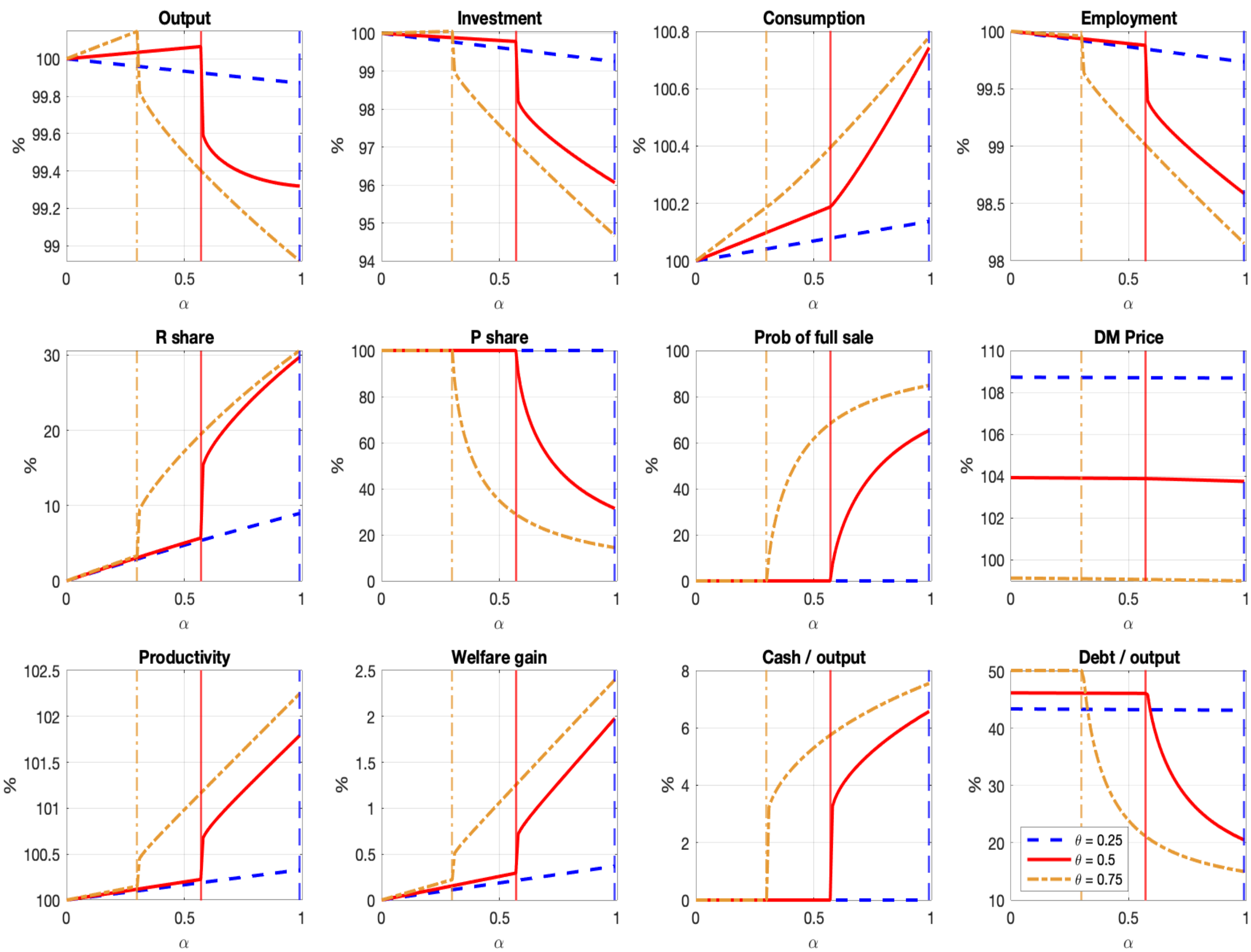
Note: For output, investment, consumption, productivity, and welfare, the corresponding levels in the economy with no reallocation are used as the normalization.

Figure 8: Long-run Effects of Search Frictions with Imperfect Credit and No Money



Note: For output, investment, consumption, productivity, and welfare, the corresponding levels in the economy with no reallocation are used as the normalization.

Figure 9: Long-run Effects of Search Frictions



Note: the vertical lines divide non-monetary (left) and monetary (right) regions. For output, investment, consumption, productivity, and welfare, the corresponding levels in the economy with no reallocation are used as the normalization.

Figure 10: Effect of τ_k

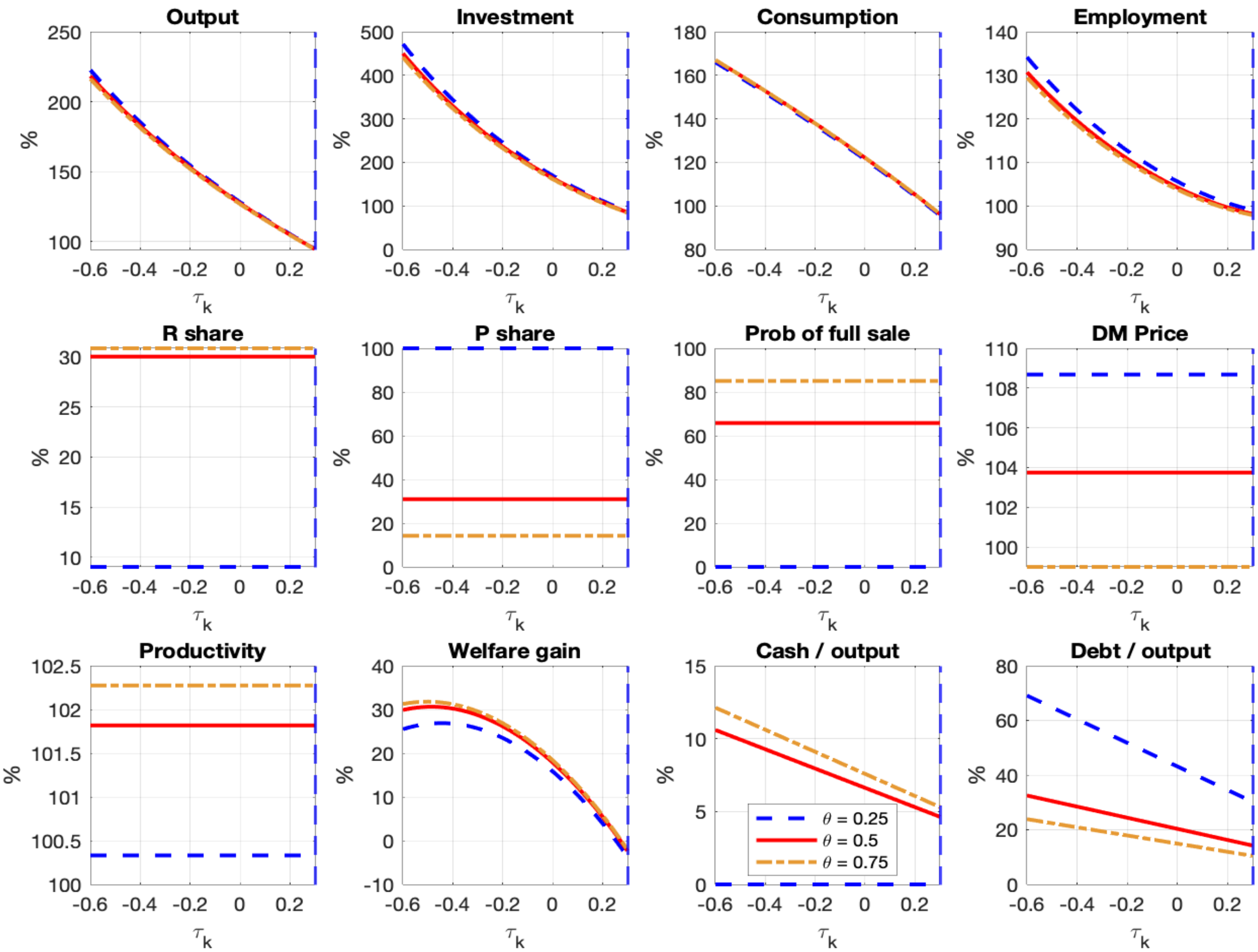


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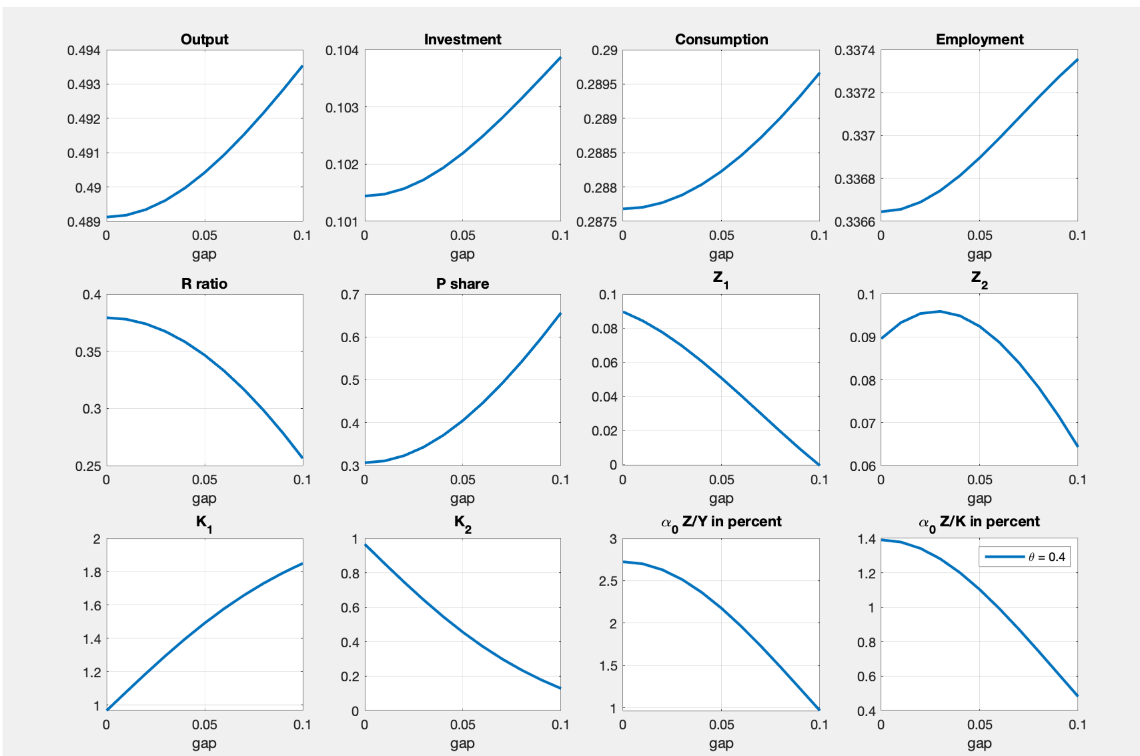
Figure 11: Effect of Differences in Persistent Component of Productivity



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