Sound and Fury: Signaling in Sovereign Debt Markets

VERY PRELIMINARY AND INCOMPLETE: DO NOT CIRCULATE

Zachary R. Stangebye* Mark L. J. Wright†

January 7, 2021

Abstract

We present a model of asymmetric information and sovereign default in which lenders infer persistent, hidden sovereign types from both borrowing and default behavior. Sovereigns come in two persistent types with different proclivities to default and borrow. Transitory liquidity shocks obscure perfect revelation. While the equilibrium exhibits separation along both the default and the borrowing margin, it also features a strong attenuation effect: The bad type receives better prices than he otherwise would, which induces him to repay more often. The reverse is true for the good type. This attenuation in default behavior implies that equilibrium beliefs, while quite volatile, matter very little for price dynamics, a phenomenon we refer to as the 'Macbeth effect.' This removes the bad type's incentive to "mimic" the good type. As a result the good type fully reveals himself via consolidation and deleverageing about 12.8% of the time.

1 Introduction

Information frictions have long been known to plague markets for international debt e.g. Cole and Kehoe (1998), Chari and Kehoe (2003), Angeletos and Werning (2006), Cole et al. (2016), Carlson and Hale (2006), Van Nieuwerburgh and Veldkamp (2009), or Gu and Stangebye (2018) to name only a few. Investors typically do not have access to all of the information that they would like to have at the time they make the investment decision, largely because the borrowing country is foreign in nature.

In this paper, we explore the quantitative impact of these frictions on pricing and sovereign behavior in an endogenous default model. In the model, there is a good type of sovereign and a bad type. The bad type is more myopic and thus has a desire to borrow more and a tendency to default more often. Types are

*University of Notre Dame
†Federal Reserve Bank of Minneapolis

persistent, but unobserved by lenders. There are also unobserved transitory liquidity shocks that further obfuscate inference. Lenders formulate beliefs, placing some weight on the possibility that the sovereign is the bad type and pricing debt issuance accordingly. To allow for more realistic quantitative dynamics, there are also output shocks observed by all agents, long-maturity debt, lender risk-aversion, and recovery.

Despite the vast literature exploring different facets of informational asymmetries, to the authors' knowledge a general recursive signaling game between a sovereign government and its lenders in which all actions can be interpreted as signals has not been studied. Default alone as a signal has been extensively studied (see Cole and Kehoe [1998], Sandleris [2008], Phan [2017a, 2017b], or Amador and Phelan [2018]). In this paper we undertake this endeavor and study a quantitative model in the vein of Arellano (2008) or Aguiar et al. (2016) in which foreign lenders do not observe certain domestic shocks, but the sovereign does. A proper signaling game is exposited in which default and fiscal behavior, e.g., borrowing both serve as signals.

Since models of asymmetric information are often wrought with multiplicity (...), we always select the Markov Perfect Bayesian Equilibrium of the model that is the limit of the finite-horizon game. In a calibration to Russia post-2010 in which we assume that types switch symmetrically every 25 years or so, we find a couple of novel and interesting implications. First, the equilibrium is separating along both margins: The policy functions governing both debt issuance and default differ markedly across types. More information is revealed after a default, though, then a debt auction: Post-default, the average absolute change in lender beliefs, in terms of the likelihood ascribed to the sovereign being the bad type, jumps by 38%; while after a debt auction this figure is 13%. This finding justifies to some extent justify the assumptions made by the literature so far that default is an important signal. But it also reveals that it is hardly the only one.

Second, we find that average belief dynamics slowly tend toward the contention that the sovereign is the good type so long as the sovereign repays, and that they shift drastically in the other direction following a default. The average change in lender beliefs that the type is bad falls by $-.2\%$ following each period of repayment, while it jumps upward by 38% following a default. These dynamics are similar to the reputational model of Amador and Phelan (2018). However, these averages mask considerable variation: The volatility of beliefs in repayment periods is 15.2%.

Third, though beliefs are quite volatile in equilibrium, we find that they matter very little for price dynamics. This is due to a strong attenuation effect that arises in general equilibrium: The price can only depend on beliefs, not the actual type. Thus, the good type and the bad type share a common price schedule conditional on beliefs. This shared price schedule is typically worse than the good type's and better than the bad type's. Thus, debt is more expensive to service for the good type and cheaper to service for the bad type, which causes the good type to default more and the bad type to default less.

In equilibrium, this implies that the two types are quite similar in their default behavior, which in turn implies that it matters little to the lenders which type of sovereign they are facing. As a result, the pricing schedule is quite flat in beliefs. Hence, beliefs can be and are quite volatile in equilibrium, but very little of it translates to price fluctuations. We refer to this as the 'Macbeth effect,' since it suggests that beliefs can be wildly volatile and truly informative, but at the end of the day contribute very little to spread volatility.

This has a further consequence for sovereign behavior: Since the price schedule is quite flat in beliefs, there is very little incentive for the bad type to mimic the good type. Indeed, the good type will sometimes fully reveal himself in repayment states by reducing his borrowing. The bad type is sufficiently myopic and the rewards in terms of better prices sufficiently small that this happens in 12.8% of repayment periods.

2 Model

2.1 Shocks and Information Structure

We begin with a model with two types of shocks: Those that are observable to all parties and those that are only observed by the sovereign. We assume that all parties are privy to realizations of the country's endowment, and that this follows a stochastic process following Arellano (2008). In particular, $z_t = \log(Y_t)$ and

$$z_t = \rho_z z_{t-1} + \sigma_z \epsilon_t$$

where ϵ_t is an iid standard normal. We succinctly write $s_t = [z_t, Y_t]$ as the vector of observable states.

In addition to these, there are two shocks unobservable to the lenders: A binary, persistent sovereign type, which follows a Markov process, and a transitory shock. We will assume that the former is the sovereign discount factor, i.e., $\beta_t \in \{\beta_L, \beta_H\}$; Aguiar et al. (2016) argue that this parameter is pivotal in governing default propensities. It is also a well-known way to influence borrowing incentives.

The transitory shock is iid and can be interpreted either as an endowment shock or a preference shock to the level of subsistence consumption, which directly affects the sovereign's incentives to borrow.[1]

We assume that $0 < \beta^L < \beta^H < 1$ and that there is a symmetric transition probability and that the process is persistent, i.e.,

$$Pr(\beta^H|\beta^L) = Pr(\beta^L|\beta^H) = p < 0.5$$

We will refer to β^L as the 'bad' type, since this type will generally suffers from more severe commitment problems than β^H. The transitory shock has any potentially type-specific distribution, $m_t \sim F_{\beta_t}$. We assume that it is continuous and iid.[2]

This dual shock structure serves a couple of purposes. First and foremost, it is empirically plausible. When lenders see a sudden surge in indebtedness or default risk, they are often left trying to tease apart how much is due to underlying institutional change in the borrowing country and how much is due to random, uncontrollable liquidity shocks that can happen even to a well-behaved government. Policy debate in the early days of nearly any debt crisis, from the Latin American debt crisis in the 1980's to the recent Eurozone crisis, often centers around these two competing frameworks.

This dual-shock structure also give the inference problem a smoothness that is conducive to computation: Minor changes in one type's behavior will typically not lead to discrete fluctuations in beliefs. This helps ensure convergence in the solution method.

We will bundle these unobserved states together into a vector $u_t = [\beta_t, m_t]$. The debt stock is observed by all parties, but for expositional clarity we do not bundle it with the other observed states.

The lenders will attempt to infer the sovereign's type from his borrowing and default behavior. We will

[1] See Chatterjee and Eyigungor (2012) for a proof of this result.

[2] Since m_t is iid, lenders will not need to carry around beliefs regarding its value. Given ρ_t, beliefs regarding it could be inferred, but they will never be used in forecasting or demand. For expositional simplicity, we omit it entirely.

assume that their beliefs at any time t are summarized by the scalar

$$\rho_t = Pr(\beta_t = \beta^L | x^t)$$

Every period, the lenders update their beliefs using Bayes' rule according to x^t, which is the lenders' entire information set at time t, which consists of the history of all endowment realizations, borrowing and default decisions, and prices.

2.2 Timing

The timing is important because beliefs can change multiple times in a given period. A sample period t takes place as follows:

1. Period t begins with the realization of observed (s_t) and unobserved (u_t) states together with some inherited debt, B_t, and current beliefs, ρ_t
2. Given $[s_t, u_t, B_t, \rho_t]$, the sovereign decides whether or not to repay B_t.
   (a) If the sovereign defaults, beliefs change to ρ_t^D via an equilibrium updating rule. The sovereign receives a default value given by $[s_t, u_t, B_t, \rho_t^D]$.[3] Following default, the sovereign will enter financial autarky so there are no other actions he can take to signal information in period t.
   (b) If the sovereign repays, beliefs change to ρ_t^R via an equilibrium updating rule. The sovereign's period t debt issuance problem thus assumes the state $[s_t, u_t, B_t, \rho_t^R]$. The choice of debt issuance further reveals information, changing ρ_t^R to ρ_t^{R+A} via an equilibrium updating rule.
3. End-of-period lender beliefs update to ρ_{t+1} from either ρ_t^D or ρ_t^{R+A} via the expected trajectory of the Markov process for β_t.

2.3 Sovereign

The sovereign borrower suffers from limited commitment. In period t, he cannot commit to either borrowing or default behavior in period $t+1$. He has a time-separable utility function, $u(\cdot)$, and is a monopolist in

[3]Debt levels will matter for the default value because the model will feature recovery.

his own debt market. Debt is long-term as in Aguiar et al. (2016), maturing at a quarterly rate λ and paying a coupon κ in each period whether or not it matures.

The sovereign will behave much as he would in a standard sovereign default model model e.g. Aguiar and Gopinath (2006) or Arellano (2008). In particular, the sovereign begins each period by making a default-repayment decision. We express the sovereign's value at period t as

$$
\begin{aligned}
V_t(s_t, u_t, \rho_t, B_t) = \max\{&V_{R,t}(s_t, u_t, \rho_t^R, B_t), V_{D,t}(s_t, u_t, \rho_t^D, B_t)\} \qquad (1)\\
\text{s.t.} \quad \rho_t^R &= G_{R,t}(s_t, \rho_t, B_t)\\
\rho_t^D &= G_{D,t}(s_t, \rho_t, B_t)
\end{aligned}
$$

where $G_{R,t}$ and $G_{D,t}$ are belief updating rules that will be described momentarily. Notice that beliefs can only be updated using publicly observed information, s_t and B_t.

The value of default depends on the current level of indebtedness for a couple of reasons. First, there is recovery, so the sovereign will eventually have to service some of it again. Second, the size of a default it may reveal additional information about the sovereign's type, and this information/reputation follows him into the period of exclusion.

We will assume as in Hatchondo and Martinez (2009) that conditional on repaying outstanding debts the sovereign chooses debt issuance from a continuous interval. This implies that his problem can be expressed recursively in a Bellman equation conditional on repayment of the current debt stock, B_t:

$$
\begin{aligned}
V_{R,t}(s_t, u_t, \rho_t^R, B_t) = \max_{B_{t+1}\in\mathcal{B}} &u\left(C_t - m_t\right) + \beta_t E_t\left[V_{t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, \rho_{t+1}, B_{t+1})\right]\\
\text{s.t.} \quad C_t &= Y_t - (\lambda+\kappa)B_t + q_t(B_{t+1}|s_t, \rho_t^R, B_t)\left[B_{t+1} - (1-\lambda)B_t\right] \qquad (2)\\
\rho_{t+1} &= p + (1-2p)G_{A,t}(s_t, \rho_t^R, B_t, B_{t+1})
\end{aligned}
$$

$G_{A,t}$ is a belief updating rule that takes into account both information revealed at issuance. Notice that belief updating also accounts for the expected trajectory of the Markov process between periods t and

$t+1$.

If the sovereign defaults he is excluded from credit markets temporarily, receiving a value $V_{D,t}$, which entails no liquidity shocks (m_t) but a state-contingent, weakly positive default cost $\psi(Y_t)$. There will be exogenous re-entry with recovery following Hatchondo et al. (2016). In particular, during the period of exclusion the sovereign faces a Poisson probability of market re-access. In order to gain re-access, when this shock arrives the sovereign must agree to continue servicing a fraction χ of the outstanding debt stock. Whether or not the sovereign agrees to this deal, the debt stock under consideration shrinks by a fraction of χ. Thus, if the first haircut deal is not accepted, the next will be smaller.

With this recovery framework, the Bellman for default is given by

$$
\begin{aligned}
V_{D,t}(s_t, u_t, \rho_t^D, B_t) &= u(Y_t - \psi(Y_t)) + \beta_t E_t \left[(1-\phi)V_{D,t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, \rho_{t+1}, B_t) + \phi V_{t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, \rho_{t+1}, \chi B_t)\right] \\
\rho_{t+1} &= p + (1-2p)\rho_t^D \qquad (3)
\end{aligned}
$$

Notice that, once in default, the type and belief process evolve exogenously with no chance for information to influence beliefs. This is because the sovereign takes no actions in default.

We also assume that the sovereign defaults if it is not possible for him to raise enough revenue to satisfy liquidity needs i.e. if $\max_{B_{t+1}} q_t(B_{t+1}|s_t, \rho_t^R, B_t)\left[B_{t+1} - (1-\lambda)B_t\right] < m_t + (\lambda+\kappa)B_t - Y_t$.

2.4 Belief Updating

In equilibrium, lenders update their beliefs regarding β every period in response to sovereign borrowing and default behavior. They do so optimally, i.e., according to Bayes' rule.

To define this, we start with the default behavior. Denote the default policy by $D_t(s_t, u_t, \rho_t, B_t)$. We define a subset of the domain of the m-shock by $M_D^i(s_t, \rho_t, B_t) = \{m | D_t(s_t, (\beta_i, m), \rho_t, B_t) = 1\}$. $G_{D,t}(s_t, \rho_t, B_t)$ describes the immediate window in which investors update their beliefs following a default: The first argument is the current beliefs were the sovereign to repay and the second is the current level of debt. Bayes' rule implies

$$G_{D,t}(s_t,\rho_t,B_t) = \begin{cases} 1, & M_D^i(s_t,\rho_t,B_t) \text{ empty for } i \in \{L,H\} \\ \frac{\rho_t \int_{m \in M_D^L(s_t,\rho_t,B_t)} f_{\beta_L}(m)dm}{\rho_t \int_{m \in M_D^L(s_t,\rho_t,B_t)} f_{\beta_L}(m)dm + (1-\rho_t)\int_{m \in M_D^H(s_t,\rho_t,B_t)} f_{\beta_H}(m)dm}, & o/w \end{cases} \tag{4}$$

Notice our required assumption regarding off-equilibrium beliefs: If the sovereign ever defaults when neither the good nor the bad type are prescribed to default, lenders will automatically assume him to be the bad type.

We can define a symmetric function for the case when the sovereign repays, noting that the repayment set over m will be the complement of the default set, i.e., $M_D^{i,C}(s_t,\rho_t,B_t)$. The only difference here will be off-equilibrium beliefs: If the sovereign ever repays when both types prescribe default for all possible unobserved states, then lenders instantly assume him to be the good type.

$$G_{R,t}(s_t,\rho_t,B_t) = \begin{cases} 0, & M_D^{i,C}(s_t,\rho_t,B_t) \text{ empty for } i \in \{L,H\} \\ \frac{\rho_t \int_{m \in M_D^{L,C}(s_t,\rho_t,B_t)} f_{\beta_L}(m)dm}{\rho_t \int_{m \in M_D^{L,C}(s_t,\rho_t,B_t)} f_{\beta_L}(m)dm + (1-\rho_t)\int_{m \in M_D^{H,C}(s_t,\rho_t,B_t)} f_{\beta_H}(m)dm}, & o/w \end{cases} \tag{5}$$

Now for the Bayesian response to borrowing. To define this, let $A_t(s_t,u_t,\rho_t^R,B_t)$ be the sovereign's borrowing policy function in equilibrium. For a given (s_t,ρ_t^R,B_t), define
$M^i(s_t,\rho_t^R,B_t,B_{t+1}) = \{m | A_t(s_t,(\beta_i,m),\rho_t^R,B_t) = B_{t+1}\}$. In states of repayment, we can define the following function:

$$G_{A,t}(s_t,\rho_t^R,B_t,B_{t+1}) = \tag{6}$$
$$\begin{cases} 1, & B_{t+1} > A_t(s_t,(\beta_i,m),\rho_t^R,B_t)\ \forall m,\ i \in \{L,H\} \\ 0, & B_{t+1} < A_t(s_t,(\beta_i,m),\rho_t^R,B_t)\ \forall m,\ i \in \{L,H\} \\ \frac{\rho_t^R \int_{m \in M^L(s_t,\rho_t^R,B_t,B_{t+1})} f_{\beta_L}(m)dm}{\rho_t^R \int_{m \in M^L(s_t,\rho_t^R,B_t,B_{t+1})} f_{\beta_L}(m)dm + (1-\rho_t^R)\int_{m \in M^H(s_t,\rho_t^R,B_t,B_{t+1})} f_{\beta_H}(m)dm}, & o/w \end{cases}$$

Notice that off-equilibrium overborrowing is instantly perceived as the bad type while off-equilibrium underborrowing is instantly perceived as the good type.

2.4.1 Off-Equilibrium Beliefs Discussion

While we cannot prove in this recursive framework that these off-equilibrium paths satisfy the intuitive criterion, it is certainly likely to be the case. The more myopic one has a greater incentive to default ceteris paribus, so it is natural that off-equilibrium default would signal the bad type. The reverse is true for repayment. As far as borrowing goes, the more myopic type suffers more from consolidation, so deleverageing ought to signal the good type and overborrowing the reverse.

2.5 Foreign Lenders

We assume that foreign lenders are competitive and risk-averse as in Aguiar et al. (2016). They arrive in overlapping generations with a wealth w_t and have access to a risk-free return of r against which they price default and dilution risk. They have a concave utility, u_L. Given these beliefs, they price default risk.

If they arrive in a state of repayment (s_t, B_t, ρ_t^R) and observe aggregate issuance is B_{t+1}, then they solve the following problem.

$$\max_{b_{t+1}} \; E_t\left[u_L(\tilde{c}_{L,t+1})\right] \tag{7}$$

where $c_{L,t+1} = (w_t - q_t b_{t+1})(1+r) + b_{t+1}\Big[\rho_{t+1} R^L(s_{t+1}, m_{t+1}, \rho_{t+1}, B_{t+1}) + (1-\rho_{t+1}) R^H(s_{t+1}, m_{t+1}, \rho_{t+1}, B_{t+1})\Big]$

$$R^i(s_{t+1}, m_{t+1}, \rho_{t+1}, B_{t+1}) = \Big(1 - D_{t+1}(s_{t+1}, (\beta^i, m_{t+1}), \rho_{t+1}, B_{t+1})\Big) \times \Big[\lambda + \kappa + (1-\lambda)\times$$
$$q_{t+1}\Big(A_{t+1}(s_{t+1}, (\beta^i, m_{t+1}), G_{R,t+1}(s_{t+1}, \rho_{t+1}, B_{t+1}), B_{t+1}) | s_{t+1}, G_{R,t+1}(s_{t+1}, \rho_{t+1}, B_{t+1}), B_{t+1})\Big)\Big] +$$
$$D_{t+1}(s_{t+1}, (\beta^i, m_{t+1}), \rho_{t+1}, B_{t+1}) \times q^D_{t+1}\Big(s_{t+1}, G_{D,t+1}(s_{t+1}, \rho_{t+1}, B_{t+1}), B_{t+1})\Big) \quad \text{for } i \in \{L, H\}$$
$$\rho_{t+1} = p + (1-2p) G_{A,t}(s_t, \rho_t^R, B_t, B_{t+1})$$

Notice we define R^i as the gross return function given a hidden type $i \in \{L, H\}$. The price q^D_{t+1} is the price of a bond that has been defaulted on. Such bonds are traded exclusively in secondary markets.

The problem for a lender who arrives in a state of default is similar but with a different gross return structure. If the initial beliefs are ρ_t (which would be ρ_t^D in the case of the first period of a default) and the defaulted-on debt stock was B_t, then the problem is.

$$\max_{b_{t+1}} \; E_t \left[u_L(\tilde{c}_{L,t+1}) \right] \tag{8}$$

$$\text{where } c_{L,t+1} = (w_t - q_t^D b_{t+1})(1+r) + b_{t+1}\Big[\rho_{t+1} R_D^L(s_{t+1}, m_{t+1}, \rho_{t+1}, B_t) + (1-\rho_{t+1}) R_D^H(s_{t+1}, m_{t+1}, \rho_{t+1}, B_t)\Big]$$

$$\begin{aligned} R_D^i(s_{t+1}, m_{t+1}, \rho_{t+1}, B_t) =& \phi\Bigg[\bigg(1 - D_{t+1}(s_{t+1}, (\beta^i, m_{t+1}), \rho_{t+1}, \chi B_t)\bigg) \times \Big[\lambda + \kappa + (1-\lambda) \times \\ & q_{t+1}\bigg(A_{t+1}(s_{t+1}, (\beta^i, m_{t+1}), G_{R,t+1}(s_{t+1}, \rho_{t+1}, \chi B_t), \chi B_t) | s_{t+1}, G_{R,t+1}(s_{t+1}, \rho_{t+1}, \chi B_t), \chi B_t)\bigg)\Big] + \\ & D_{t+1}(s_{t+1}, (\beta^i, m_{t+1}), \rho_{t+1}, \chi B_t) \times q_{t+1}^D\bigg(s_{t+1}, G_{D,t+1}(s_{t+1}, \rho_{t+1}, \chi B_t), \chi B_t)\bigg)\Bigg] + \\ & (1-\phi) q_{t+1}^D\bigg(s_{t+1}, G_{D,t+1}(s_{t+1}, \rho_{t+1}, B_t), B_t)\bigg) \quad \text{for } i \in \{L, H\} \\ \rho_{t+1} =& p + (1-2p)\rho_t \end{aligned}$$

2.6 Market Clearing

The debt market is the only active market here, so our market clearing condition is simply that

$$B_{t+1}^S = B_{t+1}^D \tag{9}$$

where B_{t+1}^S is the supply of bonds, which comes both from new auctions in the primary market ($B_{t+1} - (1-\lambda)B_t$) and from legacy bondholders in the secondary market ($(1-\lambda)B_t$). B_{t+1}^D is the demand for new bonds from incoming investors.

2.7 Equilibrium Definition

Definition 1. *A* ***Markov Perfect Bayesian Equilibrium (MPBE)*** *is a set of value, policy, belief updating, and price functions such that Equations (1)-(9) are satisfied.*

There could be many MPBE for a given parameterization. Models of asymmetric information often suffer from this problem. To select plausible equilibrium that is invariant across parameterizations, we always consider the MPBE that is the limit of the finite-horizon game as in Hatchondo and Martinez (2009).

An MPBE that induces the same policy function across types would be a pooling equilibrium, while an MPBE that induces distinct policy functions across types would be a separating equilibrium. In the former, no information is communicated from the sovereign to the lenders. In the latter, some information will be communicated, though we will not always have perfect revelation due to the presence of the transitory shocks.

2.8 Solution Method

We solve the model using a variant of the Gaussian Process Dynamic Programming algorithm described by Scheidigger and Bilionis (2019). Every relevant equilibrium function is approximated by a Gaussian Process and we employ their convergence criterion. We modify the algorithm along only a handful of dimensions:

1. Rather than draw new training inputs (grid points) every iteration, we randomly select the training inputs as a .01% typical set (see Cover and Thomas [2006]) of the unconditional distribution over states, assuming all endogenous states to be distributed uniformly. These training inputs remain fixed across all iterations.

2. We use $40 \times D$ training inputs, where D is the dimensionality of the state space.[4]

3. We employ a Matern 5/2 Kernel rather than a Square Exponential Kernel.

4. We only optimize each Gaussian Process once per iteration.[5]

All expectations are taken using a Gauss-Chebyshev quadrature with 25 nodes.

3 Quantitative Results

3.1 Calibration

We calibrate the model roughly to match Russia post-2010, a period in which they had access to credit markets but also experienced a substantial financial crisis as well as a recovery.

[4]Scheidigger and Bilionis (2019) recommend $10 \times D$ for standard macro models, but we find that many more training inputs are required to capture the high degree of non-linearity in a model of endogenous default.

[5]We found that multiple optimizations, as suggested and employed by Scheidigger and Bilionis (2019), did not facilitate faster or more reliable convergence given modifications (1)-(3).

We estimate our endowment process using MLE on Russian \$-valued GDP passed through an HP-filter with the smoothing parameter set to 1600. This gives us $\rho_z = .7225$ and $\sigma_z = .0343$, which is slightly more volatile than the literature standard of Argentina in the 1990s, but also less persistent. As a result, the unconditional volatility of the income process is very similar.

The World Bank international debt statistics reveal that the average maturity of foreign-held public and publicly guaranteed Russian debt is $\lambda = .032$, which is just slightly longer than Argentina. To total stock of this debt to GDP is 0.42. Further, the EMBI spread data for Russia at this time reveals that the average annualized spread is 2.5% while the spread volatility is 0.5%.

We set the recovery rate to the average given by Cruces and Trebesch (2013): $\chi = 0.63$. And we set the risk-free interest rate to 1% quarterly and the probability of a restructuring deal in default at $\phi = 8.3\%$ quarterly (following Mendoza and Yue [2012]).

We set $(\beta_L, \beta_H) = (0.93, 0.975)$ and set the transition probability between these at $p = 1\%$ quarterly. The sovereign and lenders both have CRRA preferences: The sovereign CRRA is set to 2.0 and the lenders is set to 5.0. Lender wealth is normalized to 1.0.

The volatility of the hidden iid liquidity shock is set to 2%, which is large enough to influence borrowing in a non-trivial way but not large enough to offset traditional income-driven dynamics.

Finally the default cost is set to the specification of Chatterjee and Eyigungor (2012): $\psi(Y) = \max\{y_0 Y + y_1 Y^2, 0.0\}$. This cost structure can be completely described by an alternate set of parameters, $(\bar{y}, MP)$, where $\bar{y}$ is the income level below which there is no cost of default and MP is the maximum proportional default cost that the sovereign can face, which is just the proportional cost at the upper bound of the income process, y_L. To begin, we set $\bar{y} = .85$ and $MP = 5.0\%$.

3.2 Simulation Analysis

This calibration generates the unconditional moments in Table 1, which are not far from the data. The default risk and associated spreads and volatility are a little high while the debt levels are a little low, but it's certainly close enough that the lessons we learn should generalize.

The behavior of beliefs is interesting. Conditional on repayment and nothing else, lenders certainly ascribe more weight to the sovereign being the good type. However, there is considerable volatility in their

Moment	Data	Model
Average Debt	42.0%	34.8%
Default Frequency	...	2.4%
Average Spread	2.5%	3.0%
Spread Volatility	0.5%	1.1%
$E[\rho_t \vert \text{Repayment}]$	...	14.3%
$E[\rho_t \vert \text{Default}]$	...	53.6%
$Std(\rho_t \vert \text{Repayment})$	...	14.8%
Full Revelation Fraction (Good Type)	...	12.8%

Table 1: Model/Data Moments

beliefs in these repayment states. In fact, the average absolute change in beliefs from period to period in repayment is 13.0%. Since this is quite close to the unconditional volatility of beliefs, it suggests that most of these belief shifts are transitory and fleeting rather than persistent and sluggish.

The dynamics of beliefs are intuitive. Defaulting is a very bad signal. Conditional on default, the weight assigned to being the bad type jumps by an average of 38.0%. Repayment has a much smaller impact on beliefs. As we mentioned above, during periods of repayment beliefs fluctuate a lot from period to period. However, on average repayment tends to foster better beliefs over time. The average change in beliefs from period to period in repayment is $-.2\%$. Thus, over long periods of time beliefs generally improve, but there is considerable transitory volatility in the interim. Defaulting then tends to shock beliefs quite severely for the worse.

Finally, notice that the good type fully reveals himself 12.8% of the time. The reason why is quite interesting and will be explored momentarily.

3.3 Policy Function Analysis

We begin with an examination of a typical slice of the pricing schedule, q_t, given in Figure 1. This figure gives the shape of the pricing function for the steady state income level at a current debt level of 0.3, which is near the ergodic mean. It gives this function when the lenders are certain the sovereign is the good type, $\rho_t = 0.0$, and when the lenders are certain that he is the bad type, $\rho_t = 1.0$. Interestingly, there is very little discrepancy in the pricing function across beliefs in equilibrium. This can be seen especially when compared to the impact of income, as in Figure 2a. This figure reveals that moderate income fluctuations have a much larger impact on the price than beliefs. This is even more surprising when we consider what

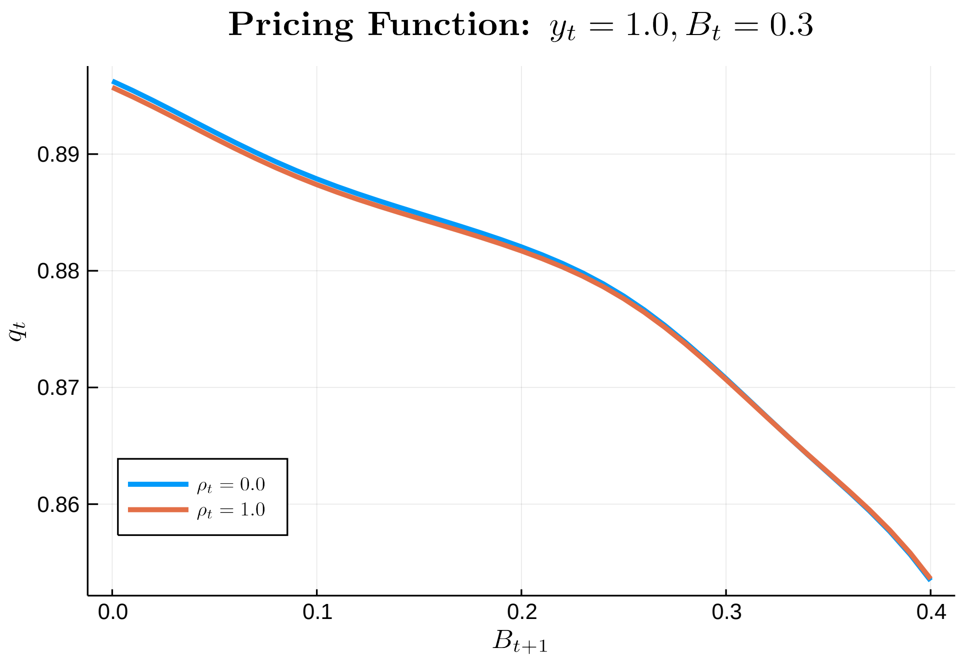


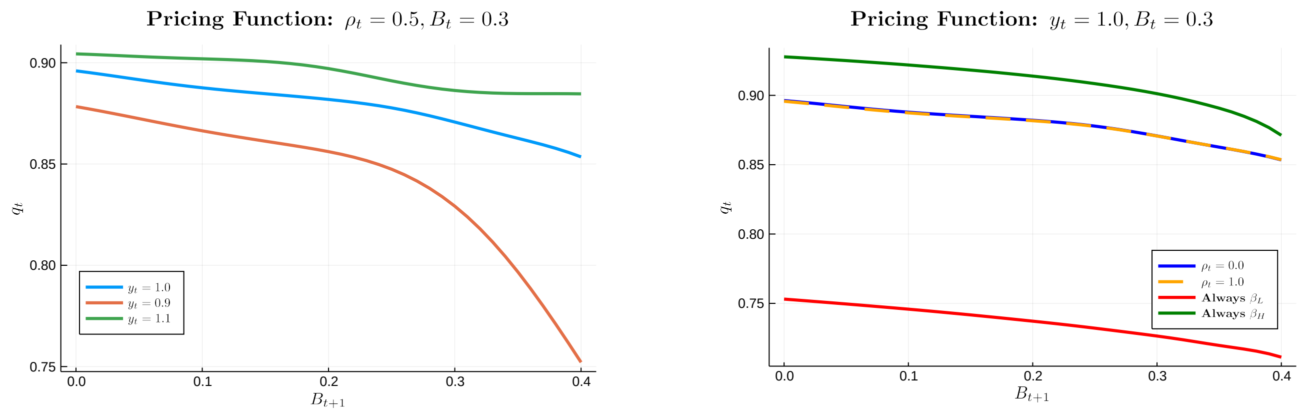
Figure 1:

sort of impact we might expect beliefs to have given types. To see this, observe Figure 2b, which compares the equilibrium pricing function to two alternate pricing functions that arise in equilibria for which the type never changes. As one would expect, the pricing schedule for the asymmetric information case is between these two. What is more astonishing is that the maximum difference across beliefs in equilibrium is orders of magnitude smaller than the difference across these two types.

This is due to a strong attenuation effect: The two types must share a pricing schedule. This price schedule will naturally be better than the bad type's and worse than the good type's, as is made clear in Figure 2b. In equilibrium, this makes it more costly for the good type to service debt and less costly for the bad type to service debt. Consequently, the good type defaults more often and the bad type default less often. This attenuates the types until they exhibit very similar default behavior. The pricing of this similar default behavior results in very little room for belief fluctuations to translate to prices in equilibrium.

This has further interesting consequences for equilibrium outcomes. First, it suggests that beliefs can (and do) fluctuate wildly in equilibrium while having little impact on prices. Price volatility is almost entirely driven by fundamental endowment fluctuations, as is the case in the standard literature.

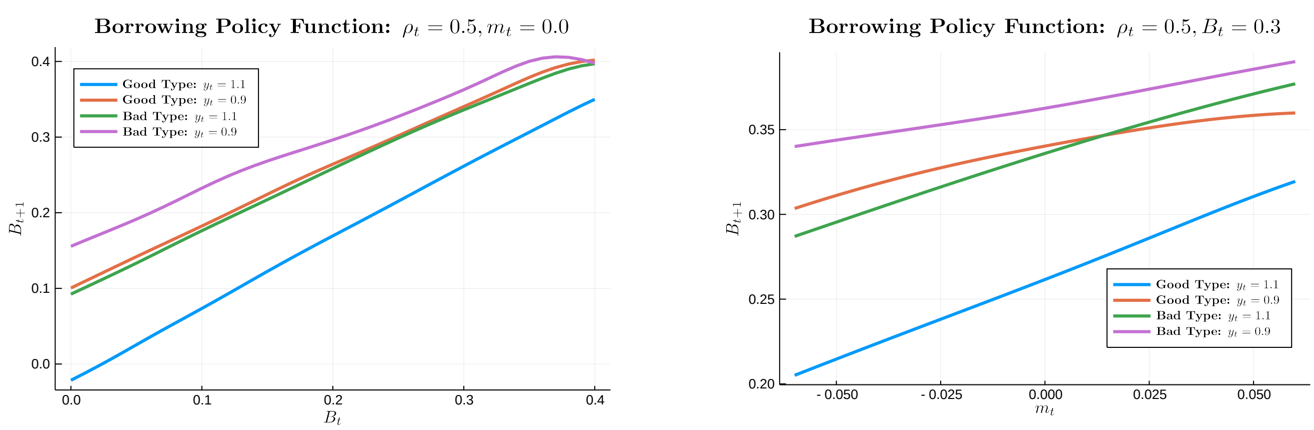
Second, it implies that the benefit to the bad type of mimicking the good type is quite low, as the



(a) **(b)**

Figure 2:

gain to be had from more favorable creditor beliefs is minor in comparison to the cost of mimicking. This is made clear in the policy functions, which can be found in Figure 3. Notice that the equilibrium is separating: The two different types do not borrow in the same capacity with respect to either observed states (here B_t) or unobserved states (here m_t). The bad type always borrows a little more, as one would expect. Figure 3b reveals further that the good type occasionally reveals himself by borrowing at levels lower than the bad type would for any level of the liquidity shock. The bad type in these regions does not find it worthwhile to mimic just for the sake obtaining better beliefs.



(a) **(b)**

Figure 3:

These differences in the policy functions manifest themselves in the belief-updating rule, G_A, as well. This can be seen in Figure 4: The more the sovereign borrows, the more likely he is to be ascribed worse beliefs. This is true regardless of the initial state.

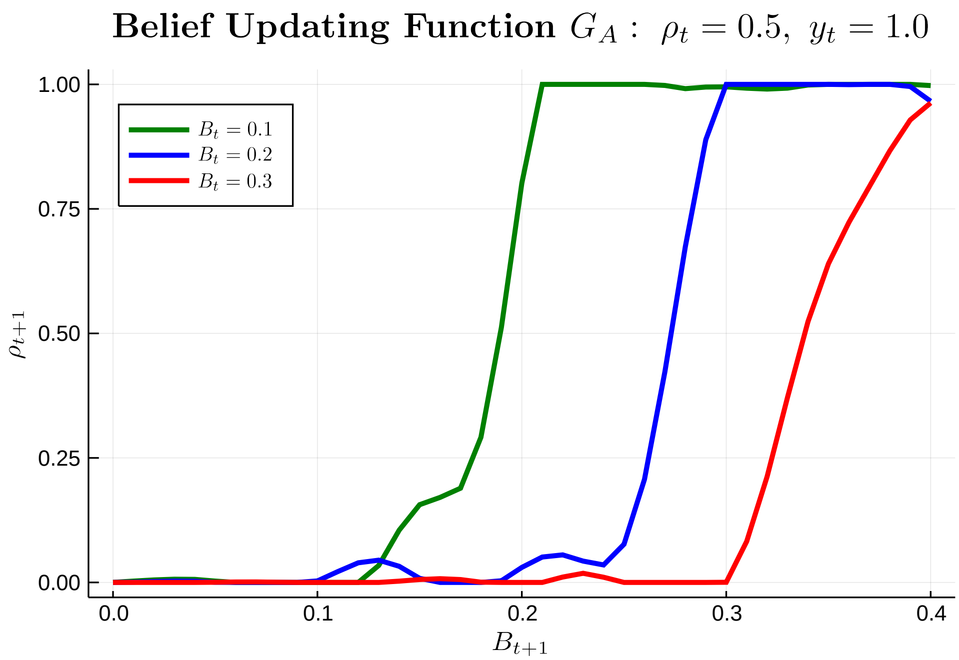


Figure 4:

4 References

References

[1] **Aguiar, Mark, Satyajit Chatterjee, Harold Cole, and Zachary R. Stangebye**, "Quantitative Models of Sovereign Debt Crises," *Handbook of Macroeconomics, Volume II*, 2016.

[2] **Amador, Manuel and Christopher Phelan**, "Repuration and Sovereign Default," *Mimeo*, 2018.

[3] **Angeletos, George-Marios and Ivan Werning**, "Crises and Prices: Information Aggregation, Multiplicity, and Volatility," *American Economic Review*, 2006, *96* (5), 1720–1736.

[4] **Arellano, Cristina**, "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 2008, *98* (3), 690–712.

[5] **Carlson, Mark and Galina B Hale**, "Rating Agencies and Sovereign Debt Rollover," *Topics in Macroeconomics*, 2006, *6* (2).

[6] **Chari, V.V. and Patrick J. Kehoe**, "Hot Money," *Journal of Political Economy*, 2003, *111* (6), 1262–1292.

[7] **Chatterjee, Satyajit and Burcu Eyigungor**, "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 2012, *102* (6), 2674–2699.

[8] **Cole, Harold L. and Patrick J. Kehoe**, "Models of Sovereign Debt: Partial Versus General Reputations," *International Economic Review*, 1998, *39* (1), 55–70.

[9] ___ **, Daniel Neuhann, and Guillermo Ordonez**, "Debt Crises: For Whom the Bell Tolls," *NBER Working Paper No. 22330*, 2016.

[10] **Gu, Grace Weishi and Zachary R. Stangebye**, "The pricing of sovereign risk under costly information," *Mimeo*, 2018.

[11] **Hatchondo, Juan Carlos and Leonardo Martinez**, "Long-Duration Bonds and Sovereign Default," *Journal of International Economics*, 2009, *79* (1), 117–125.

[12] **Phan, Toan**, "A model of sovereign debt with private information," *Journal of Economic Dynamics and Control*, 2017, *83*, 1–17.

[13] ——, "Sovereign Debt Signals," *Journal of International Economics*, 2017, *104*, 157–165.

[14] **Sandleris, Guido**, "Sovereign Defaults: Information, Investment and Credit," *Journal of International Economics*, 2008, *76* (2), 267–275.

[15] **Van Nieuwerburgh, Stijn and Laura Veldkamp**, "Information Immobility and the Home Bias Puzzle," *Journal of Finance*, 2009, *64* (3), 1187–1215.